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1.354 . . . . .	1.354
1.355 . . . . .	1.355
1.356 . . . . .	1.356
1.357 . . . . .	1.357
1.358 . . . . .	1.358
1.359 . . . . .	1.359
1.360 . . . . .	1.360
1.361 . . . . .	1.361
1.362 . . . . .	1.362
1.363 . . . . .	1.363
1.364 . . . . .	1.364
1.365 . . . . .	1.365
1.366 . . . . .	1.366
1.367 . . . . .	1.367
1.368 . . . . .	1.368
1.369 . . . . .	1.369
1.370 . . . . .	1.370
1.371 . . . . .	1.371
1.372 . . . . .	1.372
1.373 . . . . .	1.373
1.374 . . . . .	1.374
1.375 . . . . .	1.375
1.376 . . . . .	1.376
1.377 . . . . .	1.377
1.378 . . . . .	1.378
1.379 . . .	