

OXFORD MASTER SERIES IN STATISTICAL, COMPUTATIONAL, AND THEORETICAL PHYSICS

Books in this series are written for the final year undergraduate and beginning graduate level, and provide straightforward introductions to key topics in physics today. Background material and applications as well as pointers to more advanced work are included, along with ample tutorial material, examples, illustrations, chapter summaries, and graded problem sets (with some answers and hints).

Statistical mechanics is our tool for deriving the laws that emerge from complex systems. Sethna's text distills the subject to be accessible to those in all realms of science and engineering—avoiding extensive use of quantum mechanics, thermodynamics, and molecular physics. Statistical mechanics explains how bacteria search for food, and how DNA replication is proofread in biology; optimizes data compression, and explains transitions in complexity in computer science; explains the onset of chaos, and launched random matrix theory in mathematics; addresses extreme events in engineering; and models pandemics and language usage in the social sciences. Sethna's exercises introduce physicists to these triumphs and a hundred others—broadening the horizons of scholars both practicing and nascent. Flipped classrooms and remote learning can now rely on 33 pre-class exercises that test reading comprehension (Emergent vs. fundamental; Weirdness in high dimensions; Aging, entropy and DNA), and 70 in-class activities that illuminate and broaden knowledge (Card shuffling; Human correlations; Crackling noises). Science is awash in information, providing ready access to definitions, explanations, and pedagogy. Sethna's text focuses on the tools we use to create new laws, and on the fascinating simple behavior in complex systems that statistical mechanics explains.

James P. Sethna, Laboratory of Atomic and Solid State Physics, Cornell University.

'Sethna's book provides an important service to students who want to learn modern statistical mechanics. The text teaches students how to work out problems by guiding them through the exercises rather than by presenting them with worked-out examples.'

Susan Coppersmith, *Physics Today*

'The author's style, although quite concentrated, is simple to understand, and has many lovely visual examples to accompany formal ideas and concepts, which makes the exposition live and intuitively appealing.'

Olga K. Dudko, *Journal of Statistical Physics*

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1	What is statistical mechanics?	1
	Exercises	4
1.1	Quantum dice and coins	5
1.2	Probability distributions	6
1.3	Waiting time paradox	6
1.4	Stirling's formula	7
1.5	Stirling and asymptotic series	8
1.6	Random matrix theory	9
1.7	Six degrees of separation	10
1.8	Satisfactory map colorings	13
1.9	First to fail: Weibull	14
1.10	Emergence	15
1.11	Emergent vs. fundamental	15
1.12	Self-propelled particles	16
1.13	The birthday problem	17
1.14	Width of the height distribution	18
1.15	Fisher information and Cramér-Rao	19
1.16	Distances in probability space	20
2	Random walks and emergent properties	23
2.1	Random walk examples: universality and scale invariance	23
2.2	The diffusion equation	27
2.3	Currents and external forces	28
2.4	Solving the diffusion equation	30
2.4.1	Fourier	31
2.4.2	Green	31
	Exercises	33
2.1	Random walks in grade space	33
2.2	Photon diffusion in the Sun	34
2.3	Molecular motors and random walks	34
2.4	Perfume walk	35
2.5	Generating random walks	36
2.6	Fourier and Green	37
2.7	Periodic diffusion	38
2.8	Thermal diffusion	38
2.9	Frying pan	38
2.10	Polymers and random walks	38
2.11	Stocks, volatility, and diversification	39
2.12	Computational finance: pricing derivatives	40

2.13	Building a percolation network	41
2.14	Drifting random walk	43
2.15	Diffusion of nonconserved particles	44
2.16	Density dependent diffusion	44
2.17	Local conservation	44
2.18	Absorbing boundary conditions	44
2.19	Run & tumble	44
2.20	Flocking	45
2.21	Lévy flight	46
2.22	Continuous time walks: Ballistic to diffusive	47
2.23	Random walks and generating functions	48
3	Temperature and equilibrium	49
3.1	The microcanonical ensemble	49
3.2	The microcanonical ideal gas	51
3.2.1	Configuration space	51
3.2.2	Momentum space	53
3.3	What is temperature?	56
3.4	Pressure and chemical potential	59
3.5	Entropy, the ideal gas, and phase-space refinements	63
	Exercises	65
3.1	Temperature and energy	66
3.2	Large and very large numbers	66
3.3	Escape velocity	66
3.4	Pressure simulation	67
3.5	Hard sphere gas	67
3.6	Connecting two macroscopic systems	68
3.7	Gas mixture	68
3.8	Microcanonical energy fluctuations	68
3.9	Gauss and Poisson	69
3.10	Triple product relation	70
3.11	Maxwell relations	70
3.12	Solving the pendulum	71
3.13	Weirdness in high dimensions	73
3.14	Pendulum energy shell	73
3.15	Entropy maximum and temperature	74
3.16	Taste, smell, and μ	74
3.17	Undistinguished particles	75
3.18	Ideal gas glass	75
3.19	Random energy model	76
4	Phase-space dynamics and ergodicity	79
4.1	Liouville's theorem	79
4.2	Ergodicity	81
	Exercises	85
4.1	Equilibration	86
4.2	Liouville vs. the damped pendulum	86
4.3	Invariant measures	87

4.4	Jupiter! and the KAM theorem	89
4.5	No Hamiltonian attractors	91
4.6	Perverse initial conditions	91
4.7	Crooks	91
4.8	Jarzynski	93
4.9	2D turbulence and Jupiter's great red spot	94
5	Entropy	99
5.1	Entropy as irreversibility: engines and the heat death of the Universe	99
5.2	Entropy as disorder	103
5.2.1	Entropy of mixing: Maxwell's demon and osmotic pressure	104
5.2.2	Residual entropy of glasses: the roads not taken	105
5.3	Entropy as ignorance: information and memory	107
5.3.1	Nonequilibrium entropy	108
5.3.2	Information entropy	109
	Exercises	112
5.1	Life and the heat death of the Universe	113
5.2	Burning information and Maxwellian demons	113
5.3	Reversible computation	115
5.4	Black hole thermodynamics	116
5.5	Pressure-volume diagram	116
5.6	Carnot refrigerator	117
5.7	Does entropy increase?	117
5.8	The Arnol'd cat map	117
5.9	Chaos, Lyapunov, and entropy increase	119
5.10	Entropy increases: diffusion	120
5.11	Entropy of glasses	120
5.12	Rubber band	121
5.13	How many shuffles?	122
5.14	Information entropy	123
5.15	Shannon entropy	123
5.16	Fractal dimensions	124
5.17	Deriving entropy	126
5.18	Entropy of socks	126
5.19	Aging, entropy, and DNA	127
5.20	Gravity and entropy	127
5.21	Data compression	127
5.22	The Dyson sphere	129
5.23	Entropy of the galaxy	130
5.24	Nucleosynthesis and the arrow of time	130
5.25	Equilibration in phase space	133
5.26	Phase conjugate mirror	134
6	Free energies	139
6.1	The canonical ensemble	140
6.2	Uncoupled systems and canonical ensembles	143

4.4	Jupiter! and the KAM theorem	89
4.5	No Hamiltonian attractors	91
4.6	Perverse initial conditions	91
4.7	Crooks	91
4.8	Jarzynski	93
4.9	2D turbulence and Jupiter's great red spot	94
5	Entropy	99
5.1	Entropy as irreversibility: engines and the heat death of the Universe	99
5.2	Entropy as disorder	103
5.2.1	Entropy of mixing: Maxwell's demon and osmotic pressure	104
5.2.2	Residual entropy of glasses: the roads not taken	105
5.3	Entropy as ignorance: information and memory	107
5.3.1	Nonequilibrium entropy	108
5.3.2	Information entropy	109
	Exercises	112
5.1	Life and the heat death of the Universe	113
5.2	Burning information and Maxwellian demons	113
5.3	Reversible computation	115
5.4	Black hole thermodynamics	116
5.5	Pressure-volume diagram	116
5.6	Carnot refrigerator	117
5.7	Does entropy increase?	117
5.8	The Arnol'd cat map	117
5.9	Chaos, Lyapunov, and entropy increase	119
5.10	Entropy increases: diffusion	120
5.11	Entropy of glasses	120
5.12	Rubber band	121
5.13	How many shuffles?	122
5.14	Information entropy	123
5.15	Shannon entropy	123
5.16	Fractal dimensions	124
5.17	Deriving entropy	126
5.18	Entropy of socks	126
5.19	Aging, entropy, and DNA	127
5.20	Gravity and entropy	127
5.21	Data compression	127
5.22	The Dyson sphere	129
5.23	Entropy of the galaxy	130
5.24	Nucleosynthesis and the arrow of time	130
5.25	Equilibration in phase space	133
5.26	Phase conjugate mirror	134
6	Free energies	139
6.1	The canonical ensemble	140
6.2	Uncoupled systems and canonical ensembles	143

6.3	Grand canonical ensemble	146
6.4	What is thermodynamics?	147
6.5	Mechanics: friction and fluctuations	151
6.6	Chemical equilibrium and reaction rates	152
6.7	Free energy density for the ideal gas	155
	Exercises	157
6.1	Exponential atmosphere	158
6.2	Two-state system	159
6.3	Negative temperature	159
6.4	Molecular motors and free energies	160
6.5	Laplace	161
6.6	Lagrange	162
6.7	Legendre	162
6.8	Euler	162
6.9	Gibbs–Duhem	163
6.10	Clausius–Clapeyron	163
6.11	Barrier crossing	164
6.12	Michaelis–Menten and Hill	165
6.13	Pollen and hard squares	166
6.14	Statistical mechanics and statistics	167
6.15	Gas vs. rubber band	168
6.16	Rubber band free energy	169
6.17	Rubber band formalism	169
6.18	Langevin dynamics	169
6.19	Langevin simulation	170
6.20	Gibbs for pistons	171
6.21	Pistons in probability space	171
6.22	FIM for Gibbs	172
6.23	Can we burn information?	172
6.24	Word frequencies: Zipf’s law	173
6.25	Epidemics and zombies	175
6.26	Nucleosynthesis as a chemical reaction	177
7	Quantum statistical mechanics	179
7.1	Mixed states and density matrices	179
7.2	Quantum harmonic oscillator	183
7.3	Bose and Fermi statistics	184
7.4	Noninteracting bosons and fermions	185
7.5	Maxwell–Boltzmann “quantum” statistics	188
7.6	Black-body radiation and Bose condensation	190
7.6.1	Free particles in a box	190
7.6.2	Black-body radiation	191
7.6.3	Bose condensation	192
7.7	Metals and the Fermi gas	194
	Exercises	195
7.1	Ensembles and quantum statistics	195
7.2	Phonons and photons are bosons	196
7.3	Phase-space units and the zero of entropy	197

7.4	Does entropy increase in quantum systems?	198
7.5	Photon density matrices	198
7.6	Spin density matrix	198
7.7	Light emission and absorption	199
7.8	Einstein's A and B	200
7.9	Bosons are gregarious: superfluids and lasers	200
7.10	Crystal defects	201
7.11	Phonons on a string	202
7.12	Semiconductors	202
7.13	Bose condensation in a band	203
7.14	Bose condensation: the experiment	203
7.15	The photon-dominated Universe	204
7.16	White dwarfs, neutron stars, and black holes	206
7.17	Eigenstate thermalization	206
7.18	Drawing wavefunctions	206
7.19	Many-fermion wavefunction nodes	207
7.20	Cooling coffee	207
7.21	The greenhouse effect	208
7.22	Light baryon superfluids	208
7.23	Why are atoms classical?	208
7.24	Is sound a quasiparticle?	209
7.25	Quantum measurement and entropy	210
7.26	Entanglement of two spins	212
7.27	Heisenberg entanglement	213

8 Calculation and computation 217

8.1	The Ising model	217
8.1.1	Magnetism	218
8.1.2	Binary alloys	219
8.1.3	Liquids, gases, and the critical point	220
8.1.4	How to solve the Ising model	220
8.2	Markov chains	221
8.3	What is a phase? Perturbation theory	225
Exercises		227

8.1	The Ising model	228
8.2	Ising fluctuations and susceptibilities	228
8.3	Coin flips and Markov	229
8.4	Red and green bacteria	229
8.5	Detailed balance	229
8.6	Metropolis	230
8.7	Implementing Ising	230
8.8	Wolff	231
8.9	Implementing Wolff	232
8.10	Stochastic cells	232
8.11	Repressilator	234
8.12	Entropy increases! Markov chains	236
8.13	Hysteresis and avalanches	237
8.14	Hysteresis algorithms	239

8.15	NP-completeness and kSAT	240
8.16	Ising hard disks	243
8.17	Ising parallel updates	243
8.18	Ising low temperature expansion	243
8.19	2D Ising cluster expansions	244
8.20	Unicycle	244
8.21	Fruit flies and Markov	245
8.22	Metastability and Markov	246
8.23	Kinetic proofreading in cells	248
9	Order parameters, broken symmetry, and topology	253
9.1	Identify the broken symmetry	254
9.2	Define the order parameter	254
9.3	Examine the elementary excitations	258
9.4	Classify the topological defects	260
	Exercises	265
9.1	Nematic defects	265
9.2	XY defects	267
9.3	Defects and total divergences	267
9.4	Domain walls in magnets	268
9.5	Landau theory for the Ising model	269
9.6	Symmetries and wave equations	271
9.7	Superfluid order and vortices	273
9.8	Superfluids and ODLRO	274
9.9	Ising order parameter	276
9.10	Nematic order parameter	276
9.11	Pentagonal order parameter	277
9.12	Rigidity of crystals	278
9.13	Chiral wave equation	279
9.14	Sound and Goldstone's theorem	280
9.15	Superfluid second sound	281
9.16	Can't lasso a basketball	281
9.17	Fingerprints	282
9.18	Defects in crystals	284
9.19	Defect entanglement	285
9.20	Number and phase in superfluids	285
10	Correlations, response, and dissipation	287
10.1	Correlation functions: motivation	287
10.2	Experimental probes of correlations	289
10.3	Equal-time correlations in the ideal gas	290
10.4	Onsager's regression hypothesis and time correlations	292
10.5	Susceptibility and linear response	294
10.6	Dissipation and the imaginary part	295
10.7	Static susceptibility	296
10.8	The fluctuation-dissipation theorem	299
10.9	Causality and Kramers-Kronig	301

Exercises	303
10.1 Cosmic microwave background radiation	303
10.2 Pair distributions and molecular dynamics	305
10.3 Damped oscillator	307
10.4 Spin	308
10.5 Telegraph noise in nanojunctions	308
10.6 Fluctuation-dissipation: Ising	309
10.7 Noise and Langevin equations	310
10.8 Magnet dynamics	311
10.9 Quasiparticle poles and Goldstone's theorem	312
10.10 Human correlations	313
10.11 Subway bench Monte Carlo	313
10.12 Liquid free energy	315
10.13 Onsager regression hypothesis	315
10.14 Liquid dynamics	316
10.15 Harmonic susceptibility, dissipation	316
10.16 Harmonic fluctuation-dissipation	317
10.17 Susceptibilities and correlations	317
10.18 Harmonic Kramers-Kronig	318
10.19 Critical point response	318
11 Abrupt phase transitions	321
11.1 Stable and metastable phases	321
11.2 Maxwell construction	323
11.3 Nucleation: critical droplet theory	324
11.4 Morphology of abrupt transitions	326
11.4.1 Coarsening	326
11.4.2 Martensites	330
11.4.3 Dendritic growth	330
Exercises	331
11.1 Maxwell and van der Waals	332
11.2 The van der Waals critical point	332
11.3 Interfaces and van der Waals	333
11.4 Nucleation in the Ising model	333
11.5 Nucleation of dislocation pairs	334
11.6 Coarsening in the Ising model	335
11.7 Origami microstructure	336
11.8 Minimizing sequences and microstructure	338
11.9 Snowflakes and linear stability	339
11.10 Gibbs free energy barrier	341
11.11 Unstable to what?	342
11.12 Nucleation in 2D	342
11.13 Linear stability of a growing interface	342
11.14 Nucleation of cracks	343
11.15 Elastic theory does not converge	344
11.16 Mosh pits	346

12 Continuous phase transitions	349
12.1 Universality	351
12.2 Scale invariance	358
12.3 Examples of critical points	363
12.3.1 Equilibrium criticality: energy versus entropy	364
12.3.2 Quantum criticality: zero-point fluctuations versus energy	364
12.3.3 Dynamical systems and the onset of chaos	366
12.3.4 Glassy systems: random but frozen	366
12.3.5 Perspectives	367
Exercises	368
12.1 Ising self-similarity	368
12.2 Scaling and corrections to scaling	368
12.3 Scaling and coarsening	369
12.4 Bifurcation theory	369
12.5 Mean-field theory	370
12.6 The onset of lasing	371
12.7 Renormalization-group trajectories	372
12.8 Superconductivity and the renormalization group	373
12.9 Period doubling and the RG	375
12.10 RG and the central limit theorem: short	378
12.11 RG and the central limit theorem: long	378
12.12 Percolation and universality	381
12.13 Hysteresis and avalanches: scaling	383
12.14 Crackling noises	384
12.15 Hearing chaos	385
12.16 Period doubling and the onset of chaos	386
12.17 The Gutenberg–Richter law	386
12.18 Random walks and universal exponents	386
12.19 Diffusion equation and universal scaling functions	387
12.20 Hysteresis and Barkhausen noise	388
12.21 Earthquakes and wires	388
12.22 Activated rates and the saddle-node transition	389
12.23 Biggest of bunch: Gumbel	391
12.24 Extreme values: Gumbel, Weibull, and Fréchet	392
12.25 Critical correlations	393
12.26 Ising mean field derivation	394
12.27 Mean-field bound for free energy	394
12.28 Avalanche size distribution	395
12.29 The onset of chaos: lowest order RG	396
12.30 The onset of chaos: full RG	397
12.31 Singular corrections to scaling	398
12.32 Conformal invariance	399
12.33 Pandemic	402
Fourier methods	405
A.1 Fourier conventions	405
A.2 Derivatives, convolutions, and correlations	408

A.3	Fourier methods and function space	409
A.4	Fourier and translational symmetry	411
Exercises		413
A.1	Sound wave	413
A.2	Fourier cosines	413
A.3	Double sinusoid	413
A.4	Fourier Gaussians	414
A.5	Uncertainty	415
A.6	Fourier relationships	415
A.7	Aliasing and windowing	415
A.8	White noise	416
A.9	Fourier matching	417
A.10	Gibbs phenomenon	417
References		419
Index		433