

Springer Optimization and Its Applications

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Discrete Mathematics and Applications

Advances in discrete mathematics are presented in this book with applications in theoretical mathematics and interdisciplinary research. Each chapter presents new methods and techniques by leading experts. Unifying interdisciplinary applications, problems, and approaches of discrete mathematics, this book connects topics in graph theory, combinatorics, number theory, cryptography, dynamical systems, finance, optimization, and game theory. Graduate students and researchers in optimization, mathematics, computer science, economics, and physics will find the wide range of interdisciplinary topics, methods, and applications covered in this book engaging and useful.

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Automatic Sequences Are Also Non-uniformly Morphic	1
Jean-Paul Allouche and Jeffrey Shallit	
1 Introduction, Definitions, Notation	1
2 The Main Result	3
References	6
Combinatorial Identities and Inequalities for Trigonometric Sums.....	7
Horst Alzer, Omran Kouba, and Man Kam Kwong	
1 Introduction and Statement of Results	8
1.1 The Combinatorial Identity	8
1.2 Vandermonde's Convolution Formula.....	9
1.3 Harmonic Numbers	10
1.4 Trigonometric Sums.....	11
2 Proofs of Theorem 1	14
3 Proof of Theorem 2	17
4 Proof of Theorem 3	18
5 Lemmas	20
6 Proof of Theorem 4	28
7 Proof of Theorem 5	29
References	33
The Number of Partitions of a Set and Superelliptic Diophantine Equations	35
Dorin Andrica, Ovidiu Bagdasar, and George Cătălin Țurcaș	
1 Introduction	35
2 k -Partitions of Multisets with Equal Sums	38
3 k -Partitions with Equal Sums of the Set $[n]$	41
4 A Family of Diophantine Equations Defined by $Q_k(n)$	43
4.1 Proof of Theorem 6	47
4.2 The Proof of Theorem 7	50
5 Final Comments on the Family of Diophantine Equations	53
References	54

3.2	Estimation of Frequency in Three Dimensional Particle Motions .	243
3.3	Examples of Constant Time Step Exponential Integrators	245
4	Derivation of Time Adaptive Integrators Through the Geodesic Approach	248
5	Time Adaptive Exponential Variational Integrators	250
6	Numerical Results	252
6.1	Harmonic Oscillator	252
6.2	Orbits of the Two-Body Problem with Extremely High Eccentricities	254
7	Conclusions	255
	Appendix.....	256
	References	257
	Disjoint Chorded Cycles in Graphs with High Ore-Degree	259
	Alexandr Kostochka, Derrek Yager, and Gexin Yu	
1	Introduction	259
1.1	Notation.....	261
1.2	Main Result.....	261
1.3	Outline	262
2	Setup and Preliminaries.....	262
2.1	Setup	262
2.2	Preliminaries.....	263
3	Case: $G[R]$ Does Not Have a Hamiltonian Path	271
4	Case: $G[R]$ Has a Hamiltonian Path and $k \geq 3$	278
5	Case: $G[R]$ Has a Hamiltonian Path and $k = 2$	293
6	Proof of Theorem 6	302
	References	304
	A New Embedding of the $3x + 1$ Dynamical System	305
	John Leventides	
1	Introduction	305
2	The Extension $\overline{T}$ of the Collatz Map.....	309
3	The Binary Graph Arising from the Map $\overline{T}$	316
4	The Sequence of Signs $(-1)^{T^i(n)}$ and the $\overline{T}$ -Tree $G(\overline{T})$	322
5	Collatz Transition and Cyclotomy	325
6	The New Structure as a Direct System.....	329
7	Conclusions	337
	References	337
	Diffusion on Dynamical Interbank Loan Networks	339
	John Leventides and Nick Poullos	
1	Introduction	339
2	Diffusion Equations and Equilibrium Points	341
2.1	Connectivity and Equilibria.....	344
3	The Basic Two Structural Cases by Toy-Examples: Calculations of Equilibrium Points	347

3.1	Case 1: At Least One Non-zero Element per Line to the Adjacent Operator	347
3.2	Case 2: Adjacent Operator with Two Lines Equals to Zero	350
4	Differential Equation and Its Solution	351
5	Solution by Diffusion Three Structural Examples	353
5.1	Case 1: Two Real Negatives Eigenvalues and One Zero	353
5.2	Case 2: Two Complex Eigenvalues with Negative Real Part and One Zero	355
5.3	Case 3: Three Real Negative Eigenvalues	357
5.4	Solutions to the Former Three Cases Without Solving Differential Equations	360
6	Case Study in a Banking Network	361
	Reference	367

The Dynamics of Interbank Networks

John Leventides, Maria Livada, and Costas Poullos

1	Introduction	369
2	Literature Review	371
3	Interbank Networks and Default Contagion	372
4	The Bankruptcy Set of the Institution x	374
4.1	Structure of the Bankruptcy Sets $U_x, x \in \{1, 2, \dots, n\}$	375
4.2	Maximal and Minimal Elements of the Bankruptcy set U_x	375
5	The Contagion Map	378
5.1	The Contagion Graph	380
5.2	The Contagion Vector	380
6	Boolean Dynamical Systems	381
7	Fixed Points of the Function F	383
8	The Global Function	386
9	Assessment of Banks and Networks	387
9.1	Assessment of Banks	387
9.2	Assessment of Interbank Networks	388
10	Example	389
10.1	Fixed Points of the Network	391
	References	394

Prime Avoidance Property of k -th Powers of Prime Numbers with Beatty Sequence

Helmut Maier and Michael Th. Rassias

1	Introduction	397
2	Construction of the Matrix $\mathcal{M}$	400
3	Prime Numbers with Beatty Sequences	402
4	Conclusion of the Proof	402
	References	403

A Survey of Hypergraph Ramsey Problems	405
Dhruv Mubayi and Andrew Suk	
1 Introduction	405
2 General Notation	406
3 Diagonal Ramsey Numbers	406
4 Off-Diagonal Ramsey Numbers	407
5 The Erdős–Hajnal Problem.....	408
6 The Erdős–Rogers Problem	411
7 The Erdős–Gyárfás–Shelah Problem	412
8 More Off-diagonal Problems	414
8.1 $K_4^{(3)}$ Minus an Edge and a Generalization	414
8.2 Independent Neighborhoods	414
8.3 Cycles Versus Cliques.....	415
9 Bounded Degree Hypergraphs	418
10 Ordered Hypergraph Ramsey Problems	419
10.1 Tight-Paths and Cliques in Hypergraphs	419
10.2 Ordered ℓ -Power Paths in Graphs	422
11 A Bipartite Hypergraph Ramsey Problem of Erdős	423
References	424

Factorization Method for Solving Multipoint Problems for Second Order Difference Equations with Polynomial Coefficients 429

I. N. Parasidis and P. Hahamis

1 Introduction	429
2 Preliminaries	430
3 Main Results	431
4 Example Problems	435
5 Concluding Remarks.....	438
References	438

New Construction Machines of Generating Fuzzy Implications..... 441

Maria N. Rapti and Basil K. Papadopoulos

1 Introduction	441
2 Preliminaries	442
3 New Results	444
3.1 A Method of Generating Fuzzy Implications from Two Fuzzy Implications and a Fuzzy Negation	444
3.2 A Method of Generating Fuzzy Implications from Two Fuzzy Implications, a Fuzzy Negation, and an Increasing Function	448
3.3 A Method of Generating Fuzzy Implications from Two Fuzzy Implications and Two Fuzzy Negations.....	450
3.4 A Method of Generating Fuzzy Implications from Two Fuzzy Negations and an Increasing Function	452

3.5	A Method of Generating Fuzzy Implications from a t-Conorm, an Increasing Function, a Decreasing Function, and Two Fuzzy Implications.....	455
4	Conclusions	457
	References	457
	Tree Containment and Degree Conditions	459
	Maya Stein	
1	Introduction	459
2	Average Degree	461
3	Median Degree	463
4	Minimum Degree	465
5	Maximum and Minimum Degree	467
6	Expanders and Random Graphs	470
7	Ramsey Numbers	472
8	Directed Graphs.....	475
9	Hypergraphs.....	479
9.1	Tight Hypertrees.....	479
9.2	Expansions of Trees and Linear Paths.....	480
9.3	Berge Hypertrees	481
	References	482
	Extremal Singular Graphs	487
	Irene Triantafillou	
1	Introduction and Preliminaries	487
2	Extremal Graphs with Regard to Their Nullity/Rank	488
2.1	Trees	490
2.2	Bipartite Graphs	491
2.3	Unicyclic, Bicyclic, and Tricyclic Graphs	492
3	Characterization of Singular Graphs with Other Given Parameters	495
	References	497