SECOND EDITION

LATIN SQUARES AND THEIR APPLICATIONS

Donald Keedwell and József Dénes†

The second (revised) version of this book retains foundational, original material from the frequently-cited 1974 volume but is completely updated throughout. As with the earlier version, the author hopes to take the reader "from the beginnings of the subject to the frontiers of research". By omitting a few topics which were treated in the earlier edition but are no longer of current interest, the book expands upon active and emerging areas. Also, the present state of knowledge regarding the 73 then-unsolved problems given at the end of the first edition is discussed and commented upon. In addition, a number of new unsolved problems is listed.

Using an engaging narrative style, ***Latin Squares and their Applications, Second Edition***, provides thorough coverage of most parts of the subject, one of the oldest of all mathematical structures and still one of the most relevant. However, in consequence of the huge expansion of the subject in the past 40 years, some topics have had to be omitted in order to keep the book of a reasonable length.

Latin squares, or sets of mutually orthogonal latin squares (MOLS), encode the incidence structure of finite geometries; they prescribe the order in which to apply the different treatments in designing an experiment in order to permit effective statistical analysis of the results; they encapsulate the structure of finite groups and of the more general algebraic structures known as quasigroups.

† Deceased. József Dénes had endorsed the plans for this re-write but unfortunately did not live to see their implementation.

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