

Mathematics and Its History

A Concise Edition

This textbook provides a unified and concise exploration of undergraduate mathematics by approaching the subject through its history. Readers will discover the rich tapestry of ideas behind familiar topics from the undergraduate curriculum, such as calculus, algebra, topology, and more. Featuring historical episodes ranging from the Ancient Greeks to Fermat and Descartes, this volume offers a glimpse into the broader context in which these ideas developed, revealing unexpected connections that make this ideal for a senior capstone course.

The presentation of previous versions has been refined by omitting the less mainstream topics and inserting new connecting material, allowing instructors to cover the book in a one-semester course. This condensed edition prioritizes succinctness and cohesiveness, and there is a greater emphasis on visual clarity, featuring full color images and high quality 3D models. As in previous editions, a wide array of mathematical topics are covered, from geometry to computation; however, biographical sketches have been omitted.

Mathematics and Its History: A Concise Edition is an essential resource for courses or reading programs on the history of mathematics. Knowledge of basic calculus, algebra, geometry, topology, and set theory is assumed.

From reviews of previous editions:

Mathematics and Its History is a joy to read. The writing is clear, concise and inviting. The style is very different from a traditional text. I found myself picking it up to read at the expense of my usual late evening thriller or detective novel.... The author has done a wonderful job of tying together the dominant themes of undergraduate mathematics.

—Richard J. Wilders, MAA, on the Third Edition

The book...is presented in a lively style without unnecessary detail. It is very stimulating and will be appreciated not only by students. Much attention is paid to problems and to the development of mathematics before the end of the nineteenth century.... This book brings to the non-specialist interested in mathematics many interesting results. It can be recommended for seminars and will be enjoyed by the broad mathematical community.

—European Mathematical Society, on the Second Edition

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