

Scott Balchin

A Handbook of Model Categories

This book outlines a vast array of techniques and methods regarding model categories, without focussing on the intricacies of the proofs. Quillen model categories are a fundamental tool for the understanding of homotopy theory. While many introductions to model categories fall back on the same handful of canonical examples, the present book highlights a large, self-contained collection of other examples which appear throughout the literature. In particular, it collects a highly scattered literature into a single volume.

The book is aimed at anyone who uses, or is interested in using, model categories to study homotopy theory. It is written in such a way that it can be used as a reference guide for those who are already experts in the field. However, it can also be used as an introduction to the theory for novices.

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1	Introduction	1
1.1	Prerequisites	4
1.2	Outline and Reading Guide	4
1.3	Notation and Conventions	5
	Part I The Theory and Practice of Model Categories	7
2	On Quillen Model Categories	11
2.1	Model Categories	12
2.2	The Homotopy Category	16
2.3	Quillen Adjunctions and Equivalences	19
2.4	The Small Object Argument	21
2.5	Function Complexes and Framings	25
2.6	Homotopy Limits and Colimits in Model Categories	27
3	Properties	29
3.1	Cofibrantly Generated	30
3.2	Combinatorial	31
3.3	Cellular	32
3.4	Left Proper	33
3.5	Right Proper	34
3.6	Stable	35
3.7	Monoidal	36
3.8	Enriched	39
4	New Models from Old Ones	41
4.1	Left Bousfield Localization	42
4.1.1	Construction of Left Localizations	43
4.1.2	Preservation of Properties	45
4.1.3	Splitting Stable Monoidal Model Categories	47

4.2	Right Bousfield Localization	48
4.2.1	Construction of Right Localizations	48
4.2.2	Preservation of Properties	50
4.2.3	Right Bousfield Delocalization	51
4.3	Bousfield–Friedlander Localization	53
4.4	Transfer	54
4.4.1	Right Transfer	55
4.4.2	Left Transfer	58
4.5	Functor Categories	60
4.6	Reedy Model Structures	61
4.7	Diagrams of Model Categories	64
4.8	Limits of Model Categories	65
4.9	Mixed and Intermediate Models	66
4.10	Over- and Under-Categories	68
4.11	Cisinski Model Structures	70
4.12	Replacing Model Structures by More Convenient Ones	73
4.12.1	Replacing with Combinatorial Models	74
4.12.2	Replacing with Left Proper Simplicial Models	75
4.12.3	Replacing with Right Proper Models	76
4.13	Stabilization	77
4.13.1	Sequential Spectra	77
4.13.2	Symmetric Spectra	79
4.14	Categories of Enriched Categories	81
4.15	Model Structures on Pro-categories	84
4.16	G-Objects in a Model Category	86
5	Relation to $(\infty, 1)$-Categories	89
5.1	Recollections on $(\infty, 1)$ -Categories	90
5.2	Models of $(\infty, 1)$ -Categories	92
5.3	From Model Categories to $(\infty, 1)$ -Categories	94
	Part II Examples	97
6	Simplicial Sets	101
6.1	The Kan–Quillen Model	104
6.2	The Joyal Model	107
6.3	The Beke Model Structures	110
6.4	Truncated Models	111
6.5	The Minimal Cisinski Model	114
6.6	The Constructive Kan–Quillen Model	116
6.7	Model for Cylinders	117
6.8	The Category of Simplicial Algebras	120

6.9	Pointed and Reduced Simplicial Sets	122
6.10	Pro-simplicial Sets and Simplicial Pro-sets	125
	References	127
7	Topological Spaces	131
7.1	The Quillen Model Structure	132
7.2	The Strøm Model Structure	135
7.3	The Mixed Model Structure	137
7.4	Models for G -Spaces	138
7.5	Isovariant Model Structures	139
7.6	Hurewicz Models on Topological Categories	141
	References	142
8	Chain Complexes	145
8.1	Models on Bounded Chain Complexes	146
8.2	The Injective and Projective Models	148
8.3	Hurewicz and Relative Model Structures	152
8.4	Modules over Diagrams of Rings	154
8.5	Abelian Model Categories and Cotorsion Pairs	155
	References	158
9	Categories	161
9.1	The Natural Model	161
9.2	The Morita Model Structure	165
9.3	The Thomason Model	166
9.4	The $\mathcal{M}$ -Model	171
9.5	The Global Model Structure	172
9.6	A Non-cofibrantly Generated Model	174
	References	175
10	Spectra	179
10.1	Sequential Spectra	180
10.2	Symmetric Spectra	183
10.3	Orthogonal Spectra	186
10.4	Combinatorial Spectra	188
10.5	The Positive Flat Model Structure	189
	References	191
11	Simplicial Categories	195
11.1	The Bergner Model	196
11.2	Models with a Fixed Object Set	199
11.3	Internal Categories of Simplicial Sets	200
	References	202

12	Bisimplicial Sets	205
12.1	Induced Models	206
12.2	(Co)diagonal Models	207
12.3	(Complete) Segal Spaces	208
12.4	Segal Categories	212
	References	214
13	Relative Categories	217
13.1	The Barwick–Kan Model	218
13.2	The n -Fold Version	222
	References	224
14	Dendroidal Sets	227
14.1	The Operadic Model	232
14.2	Complete Segal Operads	234
14.3	The Covariant Model	237
14.4	The Picard Model	239
14.5	The Test Model	241
14.6	Open Dendroidal Sets	242
	References	244
15	Cyclic Sets	247
15.1	The Dwyer–Hopkins–Kan Model	249
15.2	The Spaliński Model	250
15.3	The Monic Models	252
15.4	The Blumberg Model	253
15.5	Cyclic Objects in a Model Category	255
15.6	Crossed Simplicial Groups	256
	References	258
16	C^*-Algebras	261
16.1	A Non-model on C^* -Algebras	262
16.2	A Model for K and KK -Theory	263
16.3	Unstable Models on C^* -Spaces	264
16.4	Stable Models on C^* -Spaces	269
16.5	Models on C^* -Categories	270
16.6	Projective Systems of C^* -Algebras	273
	References	275
17	Miscellanea	277
17.1	The Trivial Model	277
17.2	The Dual Model	278
17.3	Model Structures on the Category of Sets	279
17.4	A Model Structure on Equivalence Relations	282
17.5	Stable Module Categories	283
17.6	Model Structures on Arrow Categories	287

17.7	Model Structures for Graphs	289
17.8	Model Structures on a Poset	291
17.9	Non-existence of Limits of Model Categories	293
17.10	A Model Without Functorial Factorizations	294
17.11	Non-existence of Right Transfer	296
17.12	Non-existence of Left Bousfield Localizations	297
	References	298

Part III A Model Categorical Kunstkammer	301
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18 Die Kunstkammer	305
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Bibliography	309
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Index	323
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