

K. Erciyes

Discrete Mathematics and Graph Theory

A Concise Study Companion and Guide

The study of discrete mathematics is one of the first courses on curricula in various educational disciplines such as Computer Science, Mathematics and Engineering.

Graphs are key data structures used to represent networks, chemical structures, games etc. and are increasingly used more in various applications such as bioinformatics and the Internet. Graph theory has gone through an unprecedented growth in the last few decades both in terms of theory and implementations; hence it deserves a thorough treatment which is not adequately found in any other contemporary books on discrete mathematics, whereas about 40% of this textbook is devoted to graph theory.

Employing an algorithmic approach, this clearly structured textbook/reference presents a comprehensive review of the fundamental principles of discrete mathematics with emphasis on graph theory. It aims to be a study companion and a guide for discrete mathematics and graph theory.

Topics and features:

- Provides a detailed and concise review of the main concepts of discrete mathematics
- Presents a focus on graph theory concepts
- Surveys main algorithmic methods
- Employs algorithmic solutions to many discrete math and graph theory problems
- Includes chapter summaries, end-of-chapter review questions, numerous examples, and exercises

This unique textbook can serve as a comprehensive manual of discrete mathematics and graph theory for Computer Science or non-CS majors. In addition, its easy-to-read chapters, filled with examples, make it a highly useful reference and study aid for professionals and researchers who have not taken any discrete math course previously.

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Part I Fundamentals of Discrete Mathematics

1	Logic	3
1.1	Propositional Logic	3
1.1.1	Compound Propositions	4
1.1.2	Conditional Statements	5
1.1.3	Biconditional Statements	7
1.1.4	Tautologies and Contradictions	8
1.1.5	Equivalences	10
1.1.6	Laws of Logic	12
1.2	Predicate Logic	14
1.2.1	Quantifiers	15
1.2.2	Propositional Functions with Two Variables	16
1.2.3	Negation	17
1.2.4	The Universal Conditional Statement	18
1.2.5	The Existential Conditional Statements	19
1.3	Review Questions	20
1.4	Chapter Notes	20
2	Proofs	23
2.1	Arguments	23
2.1.1	Rules of Inference	23
2.1.2	Definitions	27
2.2	Direct Proof	28
2.3	Contrapositive	30
2.4	Proof by Contradiction	31
2.5	Proving Biconditional Propositions	32
2.6	Proofs Using Quantifiers	33
2.6.1	Proving Universal Statements	34
2.6.2	Proving Existential Statements	35
2.7	Proof by Cases	36
2.8	Review Questions	37
2.9	Chapter Notes	38

3	Algorithms	
3.1	Basics	
3.1.1	Pseudocode Convention	
3.1.2	Assignment and Types of Variables	
3.1.3	Decision	
3.1.4	Loops	
3.1.5	Functions and Parameter Passing	
3.2	Basic Data Structures	
3.3	Sorting	
3.3.1	Bubble Sort	
3.3.2	Exchange Sort	
3.4	Analysis	
3.5	Design Methods	
3.5.1	Divide and Conquer	
3.5.2	Greedy Method	
3.5.3	Dynamic Programming	
3.6	Difficult Problems	
3.7	Review Questions	
3.8	Chapter Notes	
	References	
4	Set Theory	
4.1	Definitions	
4.1.1	Equality of Sets	
4.1.2	Cardinality of a Set	
4.2	Subsets	
4.3	Venn Diagrams	
4.4	Set Operations	
4.4.1	Cartesian Product	
4.4.2	Set Partition	
4.4.3	Operation Precedence	
4.5	Laws of Set Theory	
4.6	Proving Set Equations	
4.6.1	The Element Method	
4.6.2	The Tabular Method	
4.6.3	The Algebraic Method	
4.7	Review Questions	
4.8	Chapter Notes	
	References	
5	Relations and Functions	
5.1	Relations	
5.1.1	Representations	
5.1.2	Inverse of a Relation	

7.5	Congruence	139
7.6	Representation of Integers	143
7.6.1	Binary System	143
7.6.2	Hexadecimal System	144
7.7	Introduction to Cryptography	144
7.7.1	Diffie-Hellman Protocol	146
7.7.2	RSA Protocol	147
7.8	Review Questions	148
7.9	Chapter Notes	149
	References	150
8	Counting and Probability	151
8.1	Basic Counting Methods	151
8.1.1	Principle of Inclusion-Exclusion	151
8.1.2	Additive Counting Principle	153
8.1.3	Multiplicative Counting Principle	153
8.1.4	The Pigeonhole Principle	155
8.1.5	Permutations	156
8.1.6	Combinations	158
8.2	Discrete Probability	163
8.2.1	Probability Measures	163
8.2.2	Independent Events	165
8.2.3	Conditional Probability	166
8.2.4	Tree Diagrams	167
8.2.5	Random Variables	168
8.2.6	Stochastic Processes	170
8.3	Review Questions	171
8.4	Chapter Notes	171
9	Boolean Algebras and Combinational Circuits	173
9.1	Boolean Algebras	173
9.1.1	Principle of Duality	175
9.1.2	Boolean Functions	176
9.1.3	Sum-of-Products Form	176
9.1.4	Product-of-Sums Form	178
9.1.5	Conversions	179
9.1.6	Minimization	180
9.2	Combinational Circuits	183
9.2.1	Gates	184
9.2.2	Designing Combinational Circuits	186
9.2.3	Arithmetic Circuits	190
9.3	Review Questions	193
9.4	Chapter Notes	193
	Reference	195

10	Introduction to the Theory of Computation	197
10.1	Languages	197
10.2	Finite State Machines	198
10.3	Finite State Automata	201
10.3.1	Analysis	201
10.3.2	Designing Finite State Automata	203
10.4	The Relationship Between Languages and Automata	204
10.5	Nondeterministic Finite State Automata	206
10.6	Regular Expressions	208
10.7	Turing Machines	209
10.8	Complexity Theory	212
10.8.1	Reductions	214
10.8.2	NP-Completeness	215
10.8.3	Coping with NP-Completeness	216
10.9	Review Questions	217
10.10	Chapter Notes	217
	References	218

Part II Graph Theory

11	Introduction to Graphs	221
11.1	Terminology	221
11.2	Vertex Degree	222
11.2.1	Degree Sequence	224
11.3	Directed Graph	224
11.4	Representation of a Graph	225
11.4.1	Adjacency List	225
11.4.2	Adjacency Matrix	226
11.4.3	Incidence Matrix	227
11.5	Subgraphs	227
11.6	Types of Graphs	228
11.6.1	Complete Graph	228
11.6.2	Weighted Graphs	229
11.6.3	Bipartite Graphs	229
11.6.4	Regular Graphs	230
11.6.5	Line Graphs	230
11.7	Graph Operations	231
11.7.1	Graph Union	231
11.7.2	Graph Intersection	232
11.7.3	Graph Join	232
11.7.4	Cartesian Product	233

11.8	Connectivity	234
11.8.1	Definitions	234
11.8.2	Connectedness	235
11.9	Graph Isomorphism	237
11.10	Review Questions	238
11.11	Chapter Notes	239
	References	241
12	Trees and Traversals	243
12.1	Definitions and Properties	243
12.2	Traversal Algorithms	244
12.2.1	Preorder Traversal	245
12.2.2	Postorder Traversal	245
12.3	Binary Trees	245
12.4	Binary Search Trees	247
12.5	Depth-First-Search	249
12.6	Breadth-First Search	251
12.7	Spanning Trees	253
12.7.1	Unweighted Spanning Trees	253
12.7.2	Minimum Spanning Trees	254
12.8	Review Questions	259
12.9	Chapter Notes	259
	References	262
13	Subgraphs	263
13.1	Cliques	263
13.2	Matching	264
13.2.1	Unweighted Matching	264
13.2.2	Weighted Matching	267
13.2.3	Bipartite Graph Matching	269
13.3	Independent Sets	269
13.3.1	Algorithm	269
13.4	Dominating Sets	271
13.4.1	Algorithm	273
13.5	Coloring	274
13.5.1	Vertex Coloring	274
13.5.2	Edge Coloring	275
13.6	Vertex Cover	278
13.6.1	Unweighted Vertex Cover	278
13.6.2	Weighted Vertex Cover	280
13.7	Review Questions	281
13.8	Chapter Notes	282
	References	285

14	Connectivity, Network Flows and Shortest Paths	287
14.1	Basics	287
14.1.1	Menger's Theorems	290
14.2	Connectivity Test	290
14.3	Digraph Connectivity	291
14.3.1	Strong Connectivity Check	291
14.3.2	Finding Strongly Connected Components	292
14.4	Network Flows	295
14.4.1	A Greedy Algorithm	296
14.4.2	Residual Graphs	296
14.4.3	Ford–Fulkerson Algorithm	297
14.4.4	Bipartite Graph Matching	298
14.5	Algebraic Connectivity	300
14.5.1	The Laplacian Matrix	300
14.5.2	Normalized Laplacian	301
14.5.3	Eigenvalues	301
14.6	Shortest Paths	302
14.7	Chapter Notes	304
	References	306
15	Graph Applications	307
15.1	Analysis of Large Graphs	307
15.1.1	Degree Distribution	307
15.1.2	Clustering	308
15.1.3	Matching Index	310
15.1.4	Centrality	311
15.1.5	Network Models	312
15.2	The Web	313
15.2.1	The Web Graph	314
15.2.2	Page Rank Algorithm	314
15.3	Ad hoc Wireless Networks	316
15.3.1	Routing in ad hoc Networks	317
15.3.2	Clustering and Spanning Tree Construction in a WSN	319
15.4	Biological Networks	321
15.4.1	Network Motifs	321
15.4.2	Network Alignment	322
15.5	Social Networks	323
15.5.1	Relationships	324
15.5.2	Structural Balance	324
15.6	Review Questions	325
15.7	Chapter Notes	326
	References	328