

Set Theory and Foundations of Mathematics

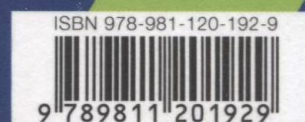
An Introduction to Mathematical Logic

Volume I Set Theory

This book provides an introduction to axiomatic set theory and descriptive set theory. It is written for the upper level undergraduate or beginning graduate students to help them prepare for advanced study in set theory and mathematical logic as well as other areas of mathematics, such as analysis, topology, and algebra.

The book is designed as a flexible and accessible text for a one-semester introductory course in set theory, where the existing alternatives may be more demanding or specialized. Readers will learn the universally accepted basis of the field, with several popular topics added as an option. Pointers to more advanced study are scattered throughout the text.

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