

Contents

Introduction	xv
Notation	xix
1 Propositional Logic	1
1.1 Boolean Functions and Formulas	2
1.2 Semantic Equivalence and Normal Forms	11
1.3 Tautologies and Logical Consequence	17
1.4 A Calculus of Natural Deduction	22
1.5 Applications of the Compactness Theorem	30
1.6 Hilbert Calculi	35
2 First-Order Logic	41
2.1 Mathematical Structures	42
2.2 Syntax of First-Order Languages	53
2.3 Semantics of First-Order Languages	61
2.4 General Validity and Logical Equivalence	73
2.5 Logical Consequence and Theories	78
2.6 Explicit Definitions—Language Expansions	85
3 Complete Logical Calculi	91
3.1 A Calculus of Natural Deduction	92
3.2 The Completeness Proof	97
3.3 First Applications: Nonstandard Models	103

3.4	ZFC and Skolem's Paradox	111
3.5	Enumerability and Decidability	117
3.6	Complete Hilbert Calculi	121
3.7	First-Order Fragments	126
3.8	Extensions of First-Order Languages	129
4	Foundations of Logic Programming	135
4.1	Term Models and Herbrand's Theorem	136
4.2	Horn Formulas	140
4.3	Propositional Resolution	143
4.4	Horn Resolution	149
4.5	Unification	152
4.6	Logic Programming	156
4.7	A Proof of the Main Theorem	166
5	Elements of Model Theory	169
5.1	Elementary Extensions	170
5.2	Complete and κ -Categorical Theories	176
5.3	The Ehrenfeucht Game	183
5.4	Embedding and Characterization Theorems	186
5.5	Model Completeness	194
5.6	Quantifier Elimination	202
5.7	Reduced Products and Ultraproducts	209
6	Incompleteness and Undecidability	215
6.1	Recursive and Primitive Recursive Functions	217
6.2	Arithmetization	226
6.3	Representability of Arithmetical Predicates	234
6.4	The Representability Theorem	243
6.5	The Theorems of Gödel, Tarski, Church	250
6.6	Transfer by Interpretation	258
6.7	The Arithmetical Hierarchy	264

7 On the Theory of Self-Reference	269
7.1 The Derivability Conditions	270
7.2 The Provable Σ_1 -Completeness	277
7.3 The Theorems of Gödel and Löb	279
7.4 The Provability Logic G	284
7.5 The Modal Treatment of Self-Reference	287
7.6 A Bimodal Provability Logic for PA	291
7.7 Modal Operators in ZFC	294
Bibliography	299
Index of Terms and Names	307
Index of Symbols	317