

Contents

Forward by <i>Martin Gardner</i>	ix
A Letter to the Student	xi
Acknowledgments	xiii
Introduction: Mathematics—A Universal Language	1

1

Mathematical Ways of Thinking 5

1. The Path of a Billiard Ball	6
2. More Billiard-Ball Mathematics	12
3. Inductive Reasoning: Finding and Extending Patterns	19
4. The Limitations of Inductive Reasoning	25
5. Deductive Reasoning: Mathematical Proof	31
6. Number Tricks and Deductive Reasoning	39
Summary and Review	45
Further Exploration	50

2

Number Sequences 59

1. Arithmetic Sequences	60
2. Geometric Sequences	67
3. The Binary Sequence	75
4. The Sequence of Squares	83
5. The Sequence of Cubes	92
6. The Fibonacci Sequence	99
Summary and Review	107
Further Exploration	113

3

Functions and Their Graphs 121

1. The Idea of a Function 122
2. Descartes and the Coordinate Graph 129
3. Functions with Line Graphs 136
4. Functions with Parabolic Graphs 145
5. More Functions with Curved Graphs 153
6. Interpolation and Extrapolation:
Guessing Between and Beyond 161
- Summary and Review 168
- Further Exploration 174

4

Large Numbers and Logarithms 183

1. Large Numbers 184
2. Scientific Notation 192
3. An Introduction to Logarithms 199
4. Decimal Logarithms 209
5. Logarithms and Scientific Notation 216
6. Exponential Functions 224
- Summary and Review 230
- Further Exploration 235

5

Symmetry and Regular Figures 245

1. Symmetry 246
2. Regular Polygons 255
3. Mathematical Mosaics 266
4. Regular Polyhedra: The Platonic Solids 276
5. Semiregular Polyhedra 289
6. Pyramids and Prisms 298
- Summary and Review 309
- Further Exploration 318

6

Mathematical Curves 327

1. The Circle and the Ellipse 328
2. The Parabola 338
3. The Hyperbola 347
4. The Sine Curve 355
5. Spirals 365
6. The Cycloid 374
- Summary and Review 384
- Further Exploration 393

7

Methods of Counting 401

1. The Fundamental Counting Principle 402
2. Permutations 413
3. More on Permutations 420
4. Combinations 428
- Summary and Review 435
- Further Exploration 441

8

The Mathematics of Chance 447

1. Probability: The Measure of Chance 448
2. Dice Games and Probability 457
3. Probabilities of Successive Events 467
4. Binomial Probability 476
5. Pascal's Triangle 486
6. The Birthday Problem: Complementary Events 495
- Summary and Review 504
- Further Exploration 511

9

An Introduction to Statistics 525

1. Organizing Data: Frequency Distributions 526
2. The Breaking of Ciphers and Codes:
An Application of Statistics 537
3. Measures of Central Tendency 546
4. Measures of Variability 554
5. Displaying Data: Statistical Graphs 564
6. Collecting Data: Sampling 574
- Summary and Review 582
- Further Exploration 591

10

Topics in Topology 601

1. The Mathematics of Distortion 602
2. The Seven Bridges of Königsberg:
An Introduction to Networks 610
3. Euler Paths 617
4. Trees 625
5. The Moebius Strip and Other Surfaces 634
- Summary and Review 641
- Further Exploration 647

Appendix: Basic Ideas and Operations 655

1. Angles and Their Measurement 656
2. The Distributive Rule 658
3. Signed Numbers 659
4. Percent 661
- Answers to Selected Exercises 663
- Index 673