

26.3.3	The bound state approximation reduces to solutions in \mathbb{R}^3	1.5.126
26.3.4	Properties of the n -particle Schrödinger ground-state hamiltonian	1.6.128
26.3.5	The initial-value problem for the Schrödinger equation	2.3.131
26	Exercises	2.3.132
6	Differential equations with variable coefficients	2.3.132
	and the corresponding theory for the linear differential equation	2.3.132
	and the linear form on vector spaces using it to obtain bounds	2.3.137
6.5	Algebraic linear problems (the n -particle Schrödinger equation)	2.3.140
6.5.1	Hilbert-space products of Hilbert spaces	2.3.140
6.5.2	Fourier	2.3.141

Contents

Preface to the Second Edition	via a uniformly symmetric potential	xv
Preface to the First Edition	in spherical coordinates	xvii
List of Symbols	in spherical coordinates	xix

Introduction

I. Basic Ideas of Hilbert Space Theory

1.	Vector Spaces	1
1.1	Vector spaces over fields of scalars	11
1.2	Linear independence of vectors	13
1.3	Dimension of a vector space	14
1.4	Isomorphism of vector spaces	16
	Exercises	17
2.	Euclidean (Pre-Hilbert) Spaces	18
2.1	Inner products on vector spaces	18
2.2	The concept of norm	20
2.3	Orthogonal vectors and orthonormal bases	21
2.4	Isomorphism of Euclidean spaces	23
	Exercises	24
3.	Metric Spaces	25
3.1	Convergence in metric spaces	25
3.2	Complete metric spaces	26
3.3	Completion of a metric space	27
	Exercises	29
4.	Hilbert Space	30
4.1	Completion of a Euclidean space	30
4.2	Separable Hilbert spaces	32

* One asterisk indicates sections in which all proofs can be skipped at a first reading.

** Two asterisks indicate sections that are introductory to original papers and research material.

4.3	l^2 spaces as examples of separable Hilbert spaces	33
4.4	Orthonormal bases in Hilbert space	36
4.5	Isomorphism of separable Hilbert spaces	41
	Exercises	43
5.	Wave Mechanics of a Single Particle Moving in One Dimension	
5.1	The formalism and its partial physical interpretation	44
5.2	The wave mechanical initial-value problem	47
5.3	Bound states of the system	49
5.4	A particle moving in a square-well potential	52
	Exercises	54
	References for Further Study	56

II. Measure Theory and Hilbert Spaces of Functions

*1.	Measurable Spaces	
1.1	Boolean algebras and σ algebras of sets	58
1.2	Boolean algebras of intervals	61
1.3	Borel sets in \mathbb{R}^n	62
1.4	Monotone classes of sets	63
	Exercises	65
*2.	Measures and Measure Spaces	
2.1	The concept of measure	66
2.2	Basic properties of measures	68
2.3	Extensions of measures and outer measures	70
2.4	Cartesian products of measure spaces	76
	Exercises	79
*3.	Measurable and Integrable Functions	
3.1	The concept of a measurable function	80
3.2	Properties of measurable functions	81
3.3	Positive-definite integrable functions	85
3.4	Real and complex integrable functions	89
3.5	Infinite sequences and sums of integrals	92
3.6	Integration on Cartesian products of measure spaces	94
	Exercises	99
4.	Spaces of Square-Integrable Functions	
4.1	Square-integrable functions	101
4.2	Hilbert spaces of square-integrable functions	103
*4.3	The separability of L^2 spaces	109
4.4	Change of variables of integration	115
	Exercises	118
5.	The Hilbert Space of Systems of n Different Particles in Wave Mechanics	
5.1	The Schrödinger equation of n -particle systems	119
5.2	The center-of-mass frame of reference	122

5.3 The bound states of n -particle systems	126
5.4 Properties of the n -particle Schrödinger operator	128
5.5 The initial-value problem	131
Exercises	132
6. Direct Sums and Tensor Products of Hilbert Spaces	
6.1 Direct sums of Euclidean spaces	132
6.2 Separability and completeness of direct sums of Hilbert spaces	134
6.3 Bilinear forms on vector spaces	137
*6.4 Algebraic tensor products of vector spaces	140
*6.5 Hilbert tensor products of Hilbert spaces	144
Exercises	147
7. The Two-Body Bound-State Problem with a Spherically Symmetric Potential	
7.1 Two particles interacting via a spherically symmetric potential	147
7.2 The equation of motion in spherical coordinates	149
7.3 Spherical harmonics on the unit sphere	151
7.4 The completeness of trigonometric functions	153
7.5 The completeness of Legendre polynomials	157
7.6 Completeness of the spherical harmonics	162
7.7 The two-body problem with a Coulomb potential	164
Exercises	169
References for Further Study	171

III. Theory of Linear Operators in Hilbert Spaces

1. Linear and Antilinear Operators on Euclidean Spaces	
1.1 Linear and antilinear transformations	172
1.2 Algebraic operations with linear transformations	174
1.3 Continuous and bounded transformations	178
1.4 Examples of bounded and unbounded operators	179
Exercises	180
2. Linear Operators in Hilbert Spaces	
2.1 Linear functionals on normed spaces	182
2.2 The dual of a Hilbert space	183
2.3 Adjoints of linear operators in Hilbert spaces	186
2.4 Bounded linear operators in Hilbert spaces	188
2.5 Dirac notation for linear operators	190
2.6 Closed operators and the graph of an operator	191
2.7 Nonexistence of unbounded everywhere-defined self-adjoint operators	193
Exercises	195
3. Orthogonal Projection Operators	
3.1 Projectors onto closed subspaces of a Hilbert space	197
3.2 Algebraic properties of projectors	200
3.3 Partial ordering of projectors	202

3.4	Projectors onto intersections and orthogonal sums of subspaces	203
*3.5	Appendix: Extensions and adjoints of closed linear operators	209
	Exercises	211
4.	Isometric and Unitary Transformations	
4.1	Isometric transformations in between Hilbert spaces	212
4.2	Unitary operators and the change of orthonormal basis	214
4.3	The Fourier-Plancherel transform	216
*4.4	Cayley transforms of symmetric operators	219
4.5	Self-adjointness of position and momentum operators in wave mechanics	224
	Exercises	226
5.	Spectral Measures	
5.1	The point spectrum of a self-adjoint operator	226
5.2	Spectral resolution of self-adjoint operators with pure point spectrum	227
5.3	Weak, strong, and uniform operator limits	229
5.4	Spectral measures and complex measures	231
5.5	Spectral functions	235
*5.6	Appendix: Signed measures	236
	Exercises	240
*6.	The Spectral Theorem for Unitary and Self-Adjoint Operators	
6.1	Spectral decomposition of a unitary operator	241
6.2	Monotonic sequences of linear operators	242
6.3	Construction of spectral families for unitary operators	243
6.4	Uniqueness of the spectral family of a unitary operator	247
6.5	Spectral decomposition of a self-adjoint operator	249
6.6	The spectral theorem for bounded self-adjoint operators	253
	Exercises	255
	References for Further Study	256

IV. The Axiomatic Structure of Quantum Mechanics

1.	Basic Concepts in the Quantum Theory of Measurement	
1.1	Observables and states in quantum mechanics	257
1.2	The concept of compatible observables	260
1.3	Born's correspondence rule for determinative measurements	261
1.4	Born's correspondence rule for preparatory measurements	264
1.5	The stochastic nature of the quantum theory of measurement	267
	Exercises	268
2.	Functions of Compatible Observables	
2.1	Fundamental and nonfundamental observables	269
2.2	Bounded functions of commuting self-adjoint operators	270
2.3	Algebras of compatible observables	274

*2.4 Unbounded functions of commuting self-adjoint operators	277
Exercises	284
3. The Schroedinger, Heisenberg, and Interaction Pictures	
3.1 The general form of the Schroedinger equation	285
3.2 The evolution operator	286
3.3 The Schroedinger picture	291
3.4 The Heisenberg picture and physical equivalence of formalisms	293
3.5 The formalism of matrix mechanics	297
3.6 The interaction picture	298
Exercises	300
4. State Vectors and Observables of Compound Systems	
4.1 Superselection rules and state vectors	301
4.2 The Hilbert space of compound systems	302
4.3 Tensor products of linear operators	303
4.4 The observables of a system of distinct particles	304
4.5 Symmetric and antisymmetric tensor products of Hilbert spaces	305
4.6 The connection between spin and statistics	306
4.7 Spin and statistics for the n -body problem	308
Exercises	310
*5. Complete Sets of Observables	
5.1 The concept of a complete set of operators	311
5.2 Cyclic vectors and complete sets of operators	315
5.3 The construction of spectral representation spaces	321
5.4 Cyclicity and maximality	324
Exercises	328
6. Canonical Commutation Relations	
6.1 The empirical significance of commutation relations	329
6.2 Representations of canonical commutation relations	331
6.3 One-parameter Abelian groups of unitary operators	334
6.4 Representations of Weyl relations	339
*6.5 Appendix: Proof of von Neumann's theorem	342
Exercises	347
7. The General Formalism of Wave Mechanics	
7.1 A derivation of one-particle wave mechanics	348
7.2 Wave mechanics of n -particle systems	351
7.3 The Schroedinger operator	354
7.4 Closures of linear operators	355
7.5 The Schroedinger kinetic energy operator	357
7.6 The Schroedinger potential energy operator	360
7.7 The self-adjointness of the Schroedinger operator	366
7.8 The angular momentum operators	369
**7.9 Time-dependent Hamiltonians	372
Exercises	373

*8.	Completely Continuous Operators and Statistical Operators	375
8.1	Completely continuous operators	375
8.2	The trace of a linear operator	380
8.3	Hilbert-Schmidt operators	383
8.4	The trace norm and the trace class	385
8.5	Statistical ensembles and the process of measurement	390
8.6	The quantum mechanical state of an ensemble	392
8.7	The von Neumann equation in Liouville space	396
8.8	Density matrices on spectral representation spaces	400
**8.9	Appendix: Classical and quantum statistical mechanics in master Liouville space	404
	Exercises	411
	References for Further Study	412

V. Quantum Mechanical Scattering Theory

1.	Basic Concepts in Scattering Theory of Two Particles	414
1.1	Scattering theory and the initial-value problem	414
1.2	Asymptotic states in classical mechanics	416
1.3	Asymptotic states and scattering states in the Schroedinger picture	418
1.4	Möller wave operators	421
1.5	The scattering operator	423
1.6	The differential scattering cross section	425
1.7	The transition operator	426
*1.8	The T -matrix formula for the differential cross section	430
	Exercises	436
2.	General Time-Dependent Two-Body Scattering Theory	438
2.1	The intertwining property of wave operators	438
2.2	The partial isometry of wave operators	440
2.3	Properties of the S operator	442
2.4	Initial and final domains of wave operators	445
2.5	Dyson's perturbation expansion	448
2.6	Criteria for existence of strong asymptotic states	451
2.7	The physical asymptotic condition	454
	Exercises	457
3.	General Time-Independent Two-Body Scattering Theory	458
3.1	The relation of the time-independent to the time-dependent approach	458
3.2	Lippmann-Schwinger equations in Hilbert space	463
3.3	Spectral integral representations of wave operators	471
3.4	The transition amplitude	472
3.5	The resolvent of an operator	474
3.6	The resolvent method in scattering theory	477
3.7	Appendix: Integration of vector- and operator-valued functions	479

**3.8	Appendix: Scattering theory in Liouville space	486
	Exercises	489
4.	Eigenfunction Expansions in Two-Body Potential Scattering Theory	
4.1	Free plane waves in three dimensions	491
4.2	Distorted plane waves	494
4.3	Free and distorted spherical waves	496
4.4	Eigenfunction expansions for complete sets of operators	498
4.5	Green's operators and Green functions	501
4.6	Lippmann-Schwinger equations for eigenfunction expansions	503
4.7	The on-shell T -matrix	506
4.8	The off-shell T -matrix and \mathcal{T} operators	510
**4.9	Appendix: Scattering theory for long-range potentials	513
*4.10	Appendix: Eigenfunctions and transition density matrices in statistical mechanics	516
	Exercises	519
5.	Green Functions in Potential Scattering	
5.1	The free Green function	520
5.2	Partial wave free Green functions	524
5.3	Fredholm integral equations with Hilbert-Schmidt kernels	525
5.4	The full Green function	528
5.5	Fredholm expansion of the full Green function	529
5.6	Symmetry properties of the full Green function	531
*5.7	Appendix: The spectrum of the Schrödinger operator	533
*5.8	Appendix: Relations between resolvents and spectral functions	539
	Exercises	542
6.	Distorted Plane Waves in Potential Scattering	
6.1	Potentials of Rollnik class	543
6.2	Fredholm series expressions for distorted plane waves	545
6.3	Asymptotic completeness and the generalized Parseval's equality	550
6.4	The scattering amplitude	553
6.5	The Born series	557
6.6	Distorted plane waves as solutions of the Schrödinger equation	560
*6.7	Appendix: Analytic operator-valued functions	564
	Exercises	567
7.	Wave and Scattering Operators in Potential Scattering	
7.1	The existence of strong asymptotic states	569
7.2	The completeness of the Möller wave operators	576
*7.3	Proof of asymptotic completeness	580
7.4	Phase shifts for scattering in central potentials	585
7.5	The general phase-shift formula for the scattering operator	588
7.6	Partial-wave analysis for spherically symmetric potentials	592
	Exercises	594
8.	Fundamental Concepts in Multichannel Scattering Theory	
8.1	The concept of channel	596

8.2	Channel Hamiltonians and wave operators	598
8.3	The uniqueness of channel strong asymptotic states	603
8.4	Interchannel scattering operators	607
8.5	The existence of strong asymptotic states in n -particle potential scattering	609
**8.6	Two-Hilbert space formulation of multichannel scattering theory	612
8.7	Multichannel eigenfunction expansions and T -matrices	615
**8.8	Multichannel Born approximations and Faddeev equations	621
*8.9	Appendix: von Neumann's mean ergodic theorem	626
	Exercises	628
References for Further Study		630
Hints and Solutions to Exercises		631
References		669
Index		679