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* One asterisk indicates sections in which all proofs can be skipped at a first reading.

** Two asterisks indicate sections that are introductory to original papers and research material.

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