

CONTENTS

[Mathematically oriented sections are marked with an asterisk *; these may be skipped without the flow of the text being lost]

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