

CONTENTS

[Mathematically oriented sections are marked with an asterisk *; these may be skipped without the flow of the text being lost]

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In the 1920s and 1930s, the pace of research had quickened exponentially over the past few years. Unfortunately, however, ideas have polarised at the same rate. Theoretical ones tend merely to reduce manipulation of mathematical equations, throwing in the occasional biological reference mostly to 'old' practical responsibility; while biologists may develop vaguely plausible deterministic models which reflect mathematical logic rather than biological reality.

Many researchers will use one approach to the total exclusion of the other, the reasons are two-fold. First, previous biological studies were greatly influenced by deterministic mathematics, and reluctance to accept stochastic ideas is still widespread. Second, too many mathematicians are content to a general solution, with the result that instead of using mathematics to describe and understand biological phenomena they become transfixed by the models themselves.

In this book we develop a unifying approach. First, we show that both deterministic and stochastic models have important roles to play and should therefore be considered together; popular deterministic ideas of logistic, elastic and predator-prey relationships can change markedly when viewed from stochastic light.

Second, in biology we are often asked to infer the nature of population development from a single dataset; yet different mechanisms of the same process can vary enormously. Since even stochastic solutions are only of limited help here, we shall construct simple computer simulation procedures which provide much needed insight into the underlying generating mechanisms. Indeed, such model-based simulations can highlight hitherto unknown features of a process and thereby suggest further profitable areas of biological investigation. Sample programs are provided to enable readers to perform their own simulation experiments.

The third approach we advocate is recognition that the environment has a