

Mathematics in Mind

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How We Understand Mathematics

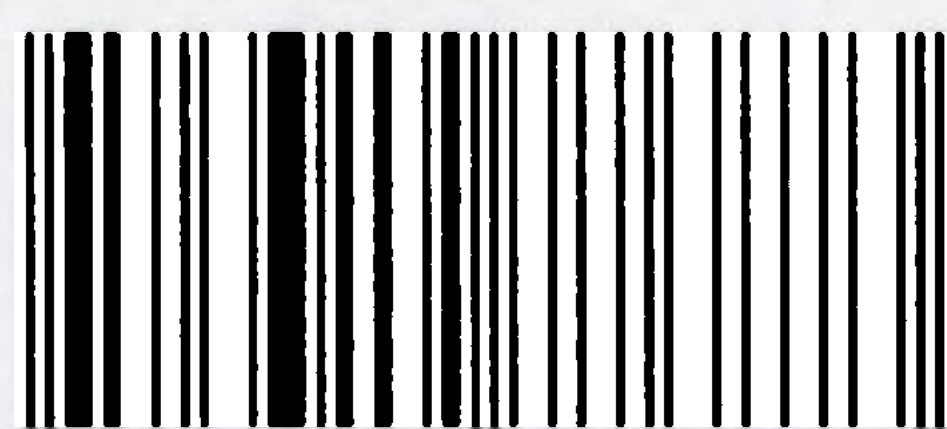
Conceptual Integration in the Language of Mathematical Description

This volume examines mathematics as a product of the human mind and analyzes the language of “pure mathematics” from various advanced-level sources. Through analysis of the foundational texts of mathematics, it is demonstrated that math is a complex literary creation, containing objects, actors, actions, projection, prediction, planning, explanation, evaluation, roles, image schemas, metonymy, conceptual blending, and, of course, (natural) language. The book follows the narrative of mathematics in a typical order of presentation for a standard university-level algebra course, beginning with analysis of set theory and mappings and continuing along a path of increasing complexity. At each stage, primary concepts, axioms, definitions, and proofs will be examined in an effort to unfold the tell-tale traces of the basic human cognitive patterns of story and conceptual blending.

This book will be of interest to mathematicians, teachers of mathematics, cognitive scientists, cognitive linguists, and anyone interested in the engaging question of how mathematics works and why it works so well.

Mathematics

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1	Introduction	1
1.1	The Effectiveness of Mathematics, Conceptual Integration, and Small Spatial Stories.	1
1.2	The Point and Method of the Book.	3
1.3	Who Is the Book Addressed To	4
1.4	The Organization of the Book	5
2	The Theoretical Framework and the Subject of Study	7
2.1	Overview.	7
2.2	Language, Cognition, and Conceptual Integration.	7
2.2.1	Cognitive Linguistics.	7
2.2.2	Conceptual Integration (Blending) Theory: The Basic Architecture	8
2.2.3	The Criticism of the Conceptual Integration Theory	11
2.2.4	The Constitutive and Governing Principles.	18
2.2.5	Small Spatial Stories and Image Schemas.	21
2.3	Modern Algebra for Beginners	25
3	Sets	31
3.1	Overview.	31
3.2	The Primitive Notions: Set and an Element	31
3.3	Subsets and Equal Sets	33
3.3.1	Subsets.	33
3.3.2	Equal Sets	34

3.4	The Null Set	38
3.5	The Union of Sets	38
3.6	The Intersection of Sets	39
3.7	Image Schemas and Small Spatial Stories for Sets and Elements.	40
3.8	Defining Sets with a Condition and Russell's Paradox	44
3.9	Proposition, Proof, and Small Spatial Stories.	47
4	Mappings.	51
4.1	Overview.	51
4.2	The Mapping as "a Carrier"	52
4.3	The "Rigorous" Definition, Ordered Pairs	52
4.4	Circularity of the "Rigorous" Definition and Conceptual Integration.	54
4.5	The Small Spatial Story of the Matchmaker	56
4.6	Definition by Graph, the Small Spatial Story of a Hiker	57
4.7	Structured Small Spatial Stories vs. Circularity of the Definition	58
5	Groups.	61
5.1	Overview.	61
5.2	The Definition of a Group and the Story of the Matchmakers	63
5.3	Abelian Groups, Finite Groups, and the Beauty of Mathematics (Part 1)	65
5.4	On the Objective Nature of Mathematics	67
5.5	The Uniqueness of the Group Elements and Conceptual Blending	68
5.6	The Force Dynamics of Mathematical Proof	70
5.7	The Subgroups, Lagrange's Theorem, and the Beauty of Mathematics (Part 2)	70
5.8	Normal Subgroups and the Beauty of Mathematics (Part 3)	74
5.9	The Homomorphism.	76
5.9.1	Homomorphism and the Carrier Story	76
5.9.2	Homomorphism and the Matchmaker Story.	78

5.9.3	Homomorphism and the Beauty of Mathematics (Part 4)	79
6	Rings, Fields, and Vector Spaces.	83
6.1	Overview	83
6.2	Definition of a Ring, Small Spatial Story of Three Matchmakers	84
6.3	The Structure of the Ring	85
6.3.1	How the Ring Matchmakers Cooperate.	85
6.3.2	Ring as a Closed Container, Force Dynamics of Proof.	88
6.4	Rings, Fields, and Arithmetic	90
6.5	From Set and Element to Arithmetic: The Story So Far	91
6.6	Multiplication by Zero Equals Zero: Proof as an Actor	92
6.7	Small Spatial Stories of Addition and Subtraction.	93
6.7.1	The Small Spatial Story of Jenga Blocks	93
6.7.2	Cayley's Theorem	94
6.7.3	The Small Spatial Story of Three Bricks.	96
6.7.4	From Bricks to Arithmetic.	98
6.7.5	Arithmetic at School vs. Modern Algebra.	100
6.8	Vector Space and the Seven Matchmakers	102
7	Summary and Conclusion	109
	Bibliography	115