

Contents

<i>Foreword by Ian Stewart</i>	xi
<i>From the Preface to the First Printing</i>	xxix
<i>From the Preface to the Seventh Printing</i>	xxxiii
<i>Preface to the Second Edition</i>	xxxv
<i>'How to Solve It' list</i>	xxxvii
<i>Introduction</i>	xxxix

PART I In the Classroom

Purpose

1. Helping the student	3
2. Questions, recommendations, mental operations	3
3. Generality	4
4. Common sense	4
5. Teacher and student. Imitation and practice	5

Main divisions, main questions

6. Four phases	6
7. Understanding the problem	7
8. Example	8
9. Devising a plan	9
10. Example	10
11. Carrying out the plan	12
12. Example	13

CONTENTS

13. Looking back	14
14. Example	15
15. Various approaches	18
16. The teacher's method of questioning	19
17. Good questions and bad questions	20

More examples

18. A problem of construction	21
19. A problem to prove	23
20. A rate problem	26

PART II

How to Solve It

A dialogue	31
------------	----

PART III

Short Dictionary of Heuristic

Analogy	39
Auxiliary elements	47
Auxiliary problem	51
Bolzano	56
Bright idea	56
Can you check the result?	58
Can you derive the result differently?	59
Can you use the result?	62
Carrying out	65
Condition	68
Contradictory [†]	69

[†] Contains only cross-references.

CONTENTS

Corollary	69
Could you derive something useful from the data?	69
Could you restate the problem? [†]	71
Decomposing and recombining	71
Definition	79
Descartes	85
Determination, hope, success	85
Diagnosis	87
Did you use all the data?	87
Do you know a related problem?	89
Draw a figure [†]	90
Examine your guess	90
Figures	94
Generalization	98
Have you seen it before?	99
Here is a problem related to yours and solved before	100
Heuristic	101
Heuristic reasoning	102
If you cannot solve the proposed problem	102
Induction and mathematical induction	103
Inventor's paradox	109
Is it possible to satisfy the condition?	109
Leibnitz	110
Lemma	110
Look at the unknown	111
Modern heuristic	116
Notation	120

[†] Contains only cross-references.

CONTENTS

Pappus	125
Pedantry and mastery	131
Practical problems	132
Problems to find, problems to prove	136
Progress and achievement	138
Puzzles	141
Reductio ad absurdum and indirect proof	142
Redundant [†]	150
Routine problem	150
Rules of discovery	150
Rules of style	151
Rules of teaching	151
Separate the various parts of the condition	151
Setting up equations	152
Signs of progress	155
Specialization	165
Subconscious work	171
Symmetry	173
Terms, old and new	173
Test by dimension	175
The future mathematician	178
The intelligent problem-solver	179
The intelligent reader	179
The traditional mathematics professor	180
Variation of the problem	181
What is the unknown?	185
Why proofs?	186

[†] Contains only cross-references.

CONTENTS

Wisdom of proverbs	191
Working backwards	194

PART IV

Problems, Hints, Solutions

Problems	206
Hints	209
Solutions	213