

Ana Hurtado, Steen Markvorsen, Maung Min-Oo and Vicente Palmer

Global Riemannian Geometry: Curvature and Topology

This book contains a clear exposition of two contemporary topics in modern differential geometry:

- distance geometric analysis on manifolds, in particular, comparison theory for distance functions in spaces which have well defined bounds on their curvature
- the study of scalar curvature rigidity and positive mass theorems using spinors and the Dirac operator

It is intended for both graduate students and researchers. This second edition has been updated to include recent developments such as promising results concerning the geometry of exit time moment spectra and potential analysis in weighted Riemannian manifolds, as well as results pertaining to an early conjecture on the geometry of the scalar curvature and speculations on new geometric approaches to the Index Theorem.

ISBN 978-3-030-55292-3



► [birkhauser-science.com](https://www.birkhauser-science.com)



Contents

Preface to the first edition, 2003	vii
Preface to the second edition, 2020	vii

1. Distance Geometric Analysis on Manifolds

Ana Hurtado, Steen Markvorsen, and Vicente Palmer

1 Appetizer and Introduction	1
2 The Comparison Setting and Preliminaries	2
3 Analysis of Riemannian Distance Functions	3
4 Analysis of Lorentzian Distance Functions	8
5 Concerning the Riemannian Setting and Notation	13
6 Green's Formulae and the Co-area Formula	16
7 The First Dirichlet Eigenvalue Comparison Theorem	17
8 Isoperimetric Relations	23
9 A Consequence of the Co-area Formula	23
10 The Fundamental Differential Equation	24
11 Isoperimetric Comparison	25
12 Mean Exit Times and Moment Spectra	29
13 The Poisson Hierarchy	32
14 Capacity Comparison	37
15 The Kelvin–Nevanlinna–Royden Criteria for Transience	42
16 Surfaces of Revolution	44
17 Warped Products	47
18 Answering the Questions in the Appetizer	49
19 Sufficient Conditions for Parabolicity and Hyperbolicity	51
20 Hyperbolicity of Spacelike Hypersurfaces	54
21 Weighted Riemannian Manifolds	56
22 Weighted Capacities	57
23 Weighted Rotationally Symmetric Spaces and the Ahlfors Criterion for Weighted Parabolicity	58
24 Weighted Curvatures	60

25	Analysis of Restricted Distance Functions in Weighted Submanifolds	61
26	Extrinsic Criteria for Weighted Parabolicity	63
27	The Grigor'yan–Fernandez Criterion for Weighted Hyperbolicity	66
28	Graphs and Flows	68
29	Scherk's Graph is Transient	71
	References	74

2. The Dirac Operator in Geometry and Physics

Maung Min-Oo

	Foreword	83
	Foreword to the Second Edition	83
1	Spinors and the Dirac Operator	84
1.1	Introduction to Spinors	84
	Examples	88
1.2	The Dirac Operator	88
1.3	The Lichnerowicz Formula	90
2	Gromov's K-Area	92
2.1	Definition of K-Area	92
2.2	The Fundamental Estimate in Terms of Scalar Curvature	95
2.3	Connections with Symplectic Invariants	101
2.4	The Vafa–Witten Inequality	102
3	Positive Mass Theorems	105
3.1	Description of Results	105
3.2	Main Ideas behind the Proofs	109
3.3	Some Mathematical Aspects of the AdS/CFT Correspondence ...	114
4	Epilogue	115
4.1	Scalar curvature on the hemisphere	116
4.2	Gromov's work on scalar curvature	117
4.3	Some speculations and musings	117
	References	118