

DISCRETE MATHEMATICS AND ITS APPLICATIONS

This book presents methods for the summation of infinite and finite series and the related identities and inversion relations. The summation includes the column sums and row sums of lower triangular matrices. The convergence of the summation of infinite series is considered.

The author's focus is on symbolic methods and the Riordan array approach. In addition, this book contains hundreds of summation formulas and identities, which can be used as a handbook for people working in computer science, applied mathematics, and computational mathematics, particularly combinatorics, computational discrete mathematics, and computational number theory. The exercises at the end of each chapter help deepen understanding.

Much of the material in this book has never appeared before in textbook form. This book can be used as a suitable textbook for advanced courses for high-level undergraduate and lower-level graduate students. It is also an introductory self-study book for researchers interested in this field, while some materials of the book can be used as a portal for further research.

$$\sum_{k \geq 0} \frac{\Delta^k \phi(t + \lambda)}{k!} =$$

MATHEMATICS



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Foreword	xi
Testimonial	xiii
Preface	xv
Biography	xix
Symbols	xxi
1 Classical Methods from Infinitesimal Calculus	1
1.1 Use of Infinitesimal Calculus	2
1.1.1 Convergence of series	2
1.1.2 Limits of sequences and series	8
1.2 Abel's Summation	20
1.2.1 Abel's theorem and Tauber theorem	20
1.2.2 Abel's summation method	27
1.3 Series Method	35
1.3.1 Use of the calculus of finite difference	36
1.3.2 Application of Euler-Maclaurin formula and the Bernoulli polynomials	48
2 Symbolic Methods	67
2.1 Symbolic Approach to Summation Formulas of Power Series	68
2.1.1 Wellknown symbolic expressions	69
2.1.2 Summation formulas related to the operator $(1 - xE)^{-1}$	74
2.1.3 Consequences and examples	77
2.1.4 Remainders of summation formulas	82
2.1.5 Q -analog of symbolic operators	86
2.2 Series Transformation	96
2.2.1 An extension of Eulerian fractions	98
2.2.2 Series-transformation formulas	99
2.2.3 Illustrative examples	105
2.3 Summation of Operators	112

2.3.1	Summation formulas involving operators	112
2.3.2	Some special convolved polynomial sums	120
2.3.3	Convolution of polynomials and two types of summations	123
2.3.4	Multifold Convolutions	125
2.3.5	Some operator summation formulas from multifold convolutions	130
3	Source Formulas for Symbolic Methods	137
3.1	An Application of Mullin-Rota's Theory of Binomial Enumeration	138
3.1.1	A substitution rule and its scope of applications . . .	138
3.1.2	Various symbolic operational formulas	141
3.1.3	Some theorems on Convergence	143
3.1.4	Examples	150
3.2	On a Pair of Operator Series Expansions Implying a Variety of Summation Formulas	156
3.2.1	A pair of (∞^4) degree formulas	157
3.2.2	Specializations and examples	160
3.2.3	A further investigation of a source formula	168
3.2.4	Various consequences of the source formula	171
3.2.5	Lifting process and formula chains	177
3.3	$(\Sigma\Delta D)$ General Source Formula and Its Applications	178
3.3.1	$(\Sigma\Delta D)$ GSF	178
3.3.2	GSF implies SF(2) and SF(3)	182
3.3.3	Embedding techniques and remarks	183
4	Methods of Using Special Function Sequences, Number Sequences, and Riordan Arrays	193
4.1	Use of Stirling Numbers, Generalized Stirling Numbers, and Eulerian Numbers	194
4.1.1	Basic convergence theorem	194
4.1.2	Summation formulas involving Stirling numbers, Bernoulli numbers, and Fibonacci numbers	200
4.1.3	Summation formulas involving generalized Eulerian functions	205
4.2	Summation of Series Involving Other Famous Number Sequences	209
4.2.1	Convergence theorem and examples	210
4.2.2	More summation formulas involving Fibonacci numbers and generalized Stirling numbers	216
4.2.3	Summation formulas of (ΣS) class	225
4.3	Summation Formulas Related to Riordan Arrays	235

4.3.1	Riordan arrays, the Riordan group, and their sequence characterizations	236
4.3.2	Identities generated by using extended Riordan arrays and Faà di Bruno's formula	247
4.3.3	Various row sums of Riordan arrays	263
4.3.4	Identities generated by using improper or non-regular Riordan arrays	272
4.3.5	Identities related to recursive sequences of order 2 and Girard-Waring identities	279
4.3.6	Summation formulas related to Fuss-Catalan numbers	285
4.3.7	One- p th Riordan arrays and Andrews' identities	295
5	Extension Methods	305
5.1	Identities and Inverse Relations Related to Generalized Riordan Arrays and Sheffer Polynomial Sequences	306
5.1.1	Generalized Riordan arrays and the recurrence relations of their entries	308
5.1.2	The Sheffer group and the Riordan group	318
5.1.3	Higher dimensional extension of the Riordan group	329
5.1.4	Dual sequences and pseudo-Riordan involutions	342
5.2	On an Extension of Riordan Array and Its Application in the Construction of Convolution-type and Abel-type Identities	354
5.2.1	Generalized Riordan arrays with respect to basic sequences of polynomials	354
5.2.2	A general class of convolution-type identities	364
5.2.3	A general class of Abel identities	374
5.3	Various Methods for constructing Identities Related to Bernoulli and Euler Polynomials	384
5.3.1	Applications of dual sequences to Bernoulli and Euler polynomials	387
5.3.2	Extended Zeilberger's algorithm for constructing identities related to Bernoulli and Euler polynomials	399
5.3.2.1	Gosper's algorithm	399
5.3.2.2	W-Z algorithm	402
5.3.2.3	Zeilberger's creative telescoping algorithm	403
5.3.2.4	Extended Zeilberger's algorithm	405
	Bibliography	417
	Index	433