

This book offers the basics of algebraic number theory for students and others who need an introduction and do not have the time to wade through the voluminous textbooks available. It is suitable for an independent study or as a textbook for a first course on the topic.

The author presents the topic here by first offering a brief introduction to number theory and a review of the prerequisite material, then presents the basic theory of algebraic numbers. The treatment of the subject is classical but the newer approach discussed at the end provides a broader theory to include the arithmetic of algebraic curves over finite fields, and even suggests a theory for studying higher dimensional varieties over finite fields. It leads naturally to the Weil conjecture and some delicate questions in algebraic geometry.

About the Author

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