

1	Geometry of complex numbers and quaternions	1
1.1	Rotations of the plane	2
1.2	Matrix representation of complex numbers	5
1.3	Quaternions	7
1.4	Consequences of multiplicative absolute value	11
1.5	Quaternion representation of space rotations	14
1.6	Discussion	18
2	Groups	23
2.1	Crash course on groups	24
2.2	Crash course on homomorphisms	27
2.3	The groups $SU(2)$ and $SO(3)$	32
2.4	Isometries of $\mathbb{R}^n$ and reflections	36
2.5	Rotations of $\mathbb{R}^4$ and pairs of quaternions	38
2.6	Direct products of groups	40
2.7	The map from $SU(2) \times SU(2)$ to $SO(4)$	42
2.8	Discussion	45
3	Generalized rotation groups	48
3.1	Rotations as orthogonal transformations	49
3.2	The orthogonal and special orthogonal groups	51
3.3	The unitary groups	54
3.4	The symplectic groups	57
3.5	Maximal tori and centers	60
3.6	Maximal tori in $SO(n)$, $U(n)$, $SU(n)$, $Sp(n)$	62
3.7	Centers of $SO(n)$, $U(n)$, $SU(n)$, $Sp(n)$	67
3.8	Connectedness and discreteness	69
3.9	Discussion	71

4	The exponential map	74
4.1	The exponential map onto $SO(2)$	75
4.2	The exponential map onto $SU(2)$	77
4.3	The tangent space of $SU(2)$	79
4.4	The Lie algebra $\mathfrak{su}(2)$ of $SU(2)$	82
4.5	The exponential of a square matrix	84
4.6	The affine group of the line	87
4.7	Discussion	91
5	The tangent space	93
5.1	Tangent vectors of $O(n)$, $U(n)$, $Sp(n)$	94
5.2	The tangent space of $SO(n)$	96
5.3	The tangent space of $U(n)$, $SU(n)$, $Sp(n)$	99
5.4	Algebraic properties of the tangent space	103
5.5	Dimension of Lie algebras	106
5.6	Complexification	107
5.7	Quaternion Lie algebras	111
5.8	Discussion	113
6	Structure of Lie algebras	116
6.1	Normal subgroups and ideals	117
6.2	Ideals and homomorphisms	120
6.3	Classical non-simple Lie algebras	122
6.4	Simplicity of $\mathfrak{sl}(n, \mathbb{C})$ and $\mathfrak{su}(n)$	124
6.5	Simplicity of $\mathfrak{so}(n)$ for $n > 4$	127
6.6	Simplicity of $\mathfrak{sp}(n)$	133
6.7	Discussion	137
7	The matrix logarithm	139
7.1	Logarithm and exponential	140
7.2	The exp function on the tangent space	142
7.3	Limit properties of log and exp	145
7.4	The log function into the tangent space	147
7.5	$SO(n)$, $SU(n)$, and $Sp(n)$ revisited	150
7.6	The Campbell–Baker–Hausdorff theorem	152
7.7	Eichler’s proof of Campbell–Baker–Hausdorff	154
7.8	Discussion	158

8	Topology	160
8.1	Open and closed sets in Euclidean space	161
8.2	Closed matrix groups	164
8.3	Continuous functions	166
8.4	Compact sets	169
8.5	Continuous functions and compactness	171
8.6	Paths and path-connectedness	173
8.7	Simple connectedness	177
8.8	Discussion	182
9	Simply connected Lie groups	186
9.1	Three groups with tangent space $\mathbb{R}$	187
9.2	Three groups with the cross-product Lie algebra	188
9.3	Lie homomorphisms	191
9.4	Uniform continuity of paths and deformations	194
9.5	Deforming a path in a sequence of small steps	195
9.6	Lifting a Lie algebra homomorphism	197
9.7	Discussion	201
	Bibliography	204
	Index	207