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1 An overview of fibred semantics and the combination of logics

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 - 1.1.2 Combining meta-level with the object level
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