

Preface

ix

Part I Foundations	1
1 Modeling the Brain	3
1-1 Introduction	4
1-2 Observations and Experimental Results Supporting Ratio Scales	7
1-3 Perception and Meaning	12
1-4 General Observations on Ratio Scales	13
1-5 Synthesis	18
1-6 Technology Needed for Validation of Mathematical Ideas	19
1-7 Confessions	20
2 The Brain	25
2-1 Introduction	26
2-2 Structure of the Brain	28
2-3 Flows in the Brain	37
2-4 Functions of the Brain	42
2-5 Purpose of the Brain	44
2-6 The Brain as a Limiting Factor	45
3 Synthesis in Hierarchies and Networks with Feedback	49
3-1 Introduction	50
3-2 Numbers Are as Good as the Scales to Which They Belong	51
3-3 Judgments and Comparisons	56
3-4 Basic Mathematical Concepts	61
3-5 Small Perturbations and Ratio Scale Approximation	70
3-6 Structural Properties of Positive Reciprocal Matrices	77
3-7 How to Structure a Hierarchy	78
3-8 Example: The Hospice Problem	79
3-9 Feedback Network	90
3-10 The Supermatrix of a Feedback System	93
3-11 The Control Hierarchy	96
3-12 Unfolding the Complexity of $f(W)$	99
3-13 How to Compute Functions of a Matrix	102
3-14 Drug-Marketing Decision	109

4	On The Shoulders of The Giants of Neuroscience	119
4-1	Introduction	120
4-2	On Neuroscience and Neuroscientists	120
4-3	Other Interesting Work in Neuroscience	127
4-4	Hierarchies and Networks in the Brain	131
4-5	Conclusion	146
5	On the Shoulders of Mathematical and Scientific Giants	151
5-1	Introduction	152
5-2	About the Origins of Some Algebraic Symbols	153
5-3	Equations, Determinants, Matrices, Eigenvalues, Eigenvectors, and Eigenfunctions	155
5-4	The Fourier Transform, Complex Variables, Distributions, and Functional Equations	167
5-5	On the Shoulders of Science and Engineering Giants	172
5-6	Conclusion	174
6	Modeling the Functions of the Brain	177
6-1	Introduction	178
6-2	Principles for Modeling the Functions of the Brain	178
6-3	The Control Hierarchy and the Basal Ganglia and Cerebellum ..	182
6-4	About Order	184
6-5	Insights about Brain Activity as Expressed in Mathematical Terms	188
Part II Theory		191
7	The Mathematics of Neurons and Signals	195
7-1	Electrical Signals in Neurons	196
7-2	Oscillatory Phenomena and Their Representation	199
7-3	Complex Variables	205
7-4	Historical Background of Trigonometric and Fourier Series	217
7-5	Fourier Series and The Fourier Transform	219
7-6	Generalized Functions and Distributions	233
7-7	Approximation and Interpolation	242
7-8	Wavelets and the Wavelet Transform	246

8	The Fundamental Formula of Relative Measurement	257
8-1	Introduction	258
8-2	Background	258
8-3	Solution of the Homogeneous Equation	265
8-4	The Logarithmic Transformation $u=\log s/\log a, a>1$	271
8-5	The Inverse Square Law of Optics, Gravitation, and Electric Charges	272
8-6	Fredholm's Inhomogeneous Equation	276
8-7	Inconsistent Kernels	281
9	Analysis of the Firing of a Single Neuron	285
9-1	Introduction	286
9-2	Derivation of the Model from First Principles	289
9-3	Mathematical Preliminaries	293
9-4A	The General Single and Multivalued Solution of $w(az)-bw(z)$	295
9-4B	The Fourier Transform is Impulsive	295
9-5A	The Analytic Solution	296
9-5B	The Fourier Transform of the Analytic Solution is Impulsive	299
9-6A	General Single-Valued Solution in the Complex Plane	300
9-6B	The Fourier Transform	302
9-7	Transformation of the Complex Domain of Firing of a Neuron to Two Periodic Real Variables	302
9-8	Tabular Summary	308
9-9	The Probability Distribution of the Number of Firings of a Single Neuron	313
10	Synthesis of the Firing of Many Neurons	321
10-1	Introduction	322
10-2	The Firing of Many Neurons: Composition and Synthesis	326
10-3	Several Ratio Scales and Related Functional Equations	327
10-4	Linear Combinations	330
10-5	Linear Combinations and Density: The Mind is the Child of the Brain	333

11	Synthesis of the Firing of All Neurons:	
	The Hypermatrix of the Neural Network Process	349
11-1	Introduction	350
11-2	Structure, Parts, and Connections	354
11-3	More about Brain Functions	358
11-4	General Description of Connections - Hierarchies and Networks	360
11-5	Magnitudes and Numbers	361
11-6	Decision Making and Prioritization - Control Function	362
11-7	Modulation-Why Many Neurons, Types of Neurons, Neurotransmitters and Neuromodulators and an Electric Field ..	370
11-8	Mathematical Representation - The Neural Hypermatrix and its Application	373
11-9	Synthesis	381
11-10	Chaos and Order - Judgment as Part of the Mainstream of Mathematics	392
11-11	Implementation - A Need for Technology	403
12	Implementing Neural Firing: Towards a new Technology	409
12-1	Introduction	410
12-2	A Neural Network	411
12-3	Sequential Firings of a Neuron	416
12-4	Data Collection	417
12-5	Neural Representation	422
12-6	On Plotting Complex Valued Functions	429
12-7	A Relief Plot of a Complex Valued Damped Periodic Function ..	430
12-8	Summary and Conclusions	431
13	Overview	433
13-1	Introduction	434
13-2	The Systems View	437
13-3	Functions and Attributes of the Brain	439
13-4	A Quick Glance at Why Mathematics is Inherent in the Structure of the Brain	448
13-5	Conclusion	451
Index	461