

Contents

Preface	vii
List of Figures	xv
1 Graphs	1
1.1 Sets, Binary Relations and Graphs	1
1.2 Some Definitions	4
1.3 Degree	11
2 Walks, Paths and Cycles	15
2.1 Basic Ideas	15
2.2 Weights and Shortest Paths	19
2.3 Euler Walks	23
2.4 Hamilton Cycles	26
2.5 The Traveling Salesman Problem	31
3 Cuts and Connectivity	35
3.1 Cutpoints and Bridges	35
3.2 Blocks	37
3.3 Connectivity	40
4 Trees	43
4.1 Characterizations of Trees	43
4.2 Spanning Trees	46
4.3 Minimal Spanning Trees	51

5	Linear Spaces Associated with Graphs	55
5.1	Finite Fields and Vector Spaces	55
5.2	The Power Set as a Vector Space	56
5.3	The Vector Spaces Associated with a Graph	58
5.4	The Cutset Subspace	60
5.5	Bases and Spanning Trees	63
6	Factorizations	69
6.1	One-Factorizations	69
6.2	Tournament Applications of One-Factorizations	75
6.3	A General Existence Theorem	77
6.4	Graphs Without One-Factors	81
7	Graph Colorings	85
7.1	Vertex Colorings	85
7.2	Brooks' Theorem	89
7.3	Counting Vertex Colorings	91
7.4	Edge-Colorings	96
7.5	Class 2 Graphs	99
8	Planarity	105
8.1	Representations and Crossings	105
8.2	Euler's Formula	108
8.3	Maps, Graphs and Planarity	111
9	Ramsey Theory	115
9.1	The Graphical Case of Ramsey's Theorem	115
9.2	Ramsey Multiplicity	120
9.3	Application of Sum-Free Sets	123
9.4	Bounds on Classical Ramsey Numbers	125
9.5	The General Case of Ramsey's Theorem	129
10	Digraphs	131
10.1	Basic Ideas	131
10.2	Orientations and Tournaments	135
10.3	Directed Euler Walks	139
11	Critical Paths	143
11.1	Activity Digraphs	143
11.2	Critical Path Analysis	146
11.3	Critical Paths Under Uncertainty	153
12	Flows in Networks	159
12.1	Transportation Networks and Flows	159
12.2	Maximal Flows	165
12.3	The Max Flow Min Cut Theorem	171

12.4	The Max Flow Min Cut Algorithm	173
12.5	Supply and Demand Problems	179
13	Computational Considerations	185
13.1	Computation Time	185
13.2	Data Structures	188
13.3	Some Graph Algorithms	190
13.4	Intractability	194
	References	197
	Hints	205
	Answers and Solutions	207
	Index	225

1.1	Diagrams of binary relations	2
1.2	Graphical representation of a road network	3
1.3	Three representations of K_4	5
1.4	Adding and deleting edges	6
1.5	A graph and its clique graph	8
1.6	$K_{4,3}$	8
1.7	The cartesian product of two graphs	9
1.8	Constructing a graph with given degrees	13
2.1	P_k and C_k	17
2.2	Find all paths from s to t	17
2.3	Petersen graph	18
2.4	Find the path of minimum weight from s to t	21
2.5	Graphs for Exercise 2.2.2	22
2.6	Graphs for Exercise 2.2.3	22
2.7	Königsberg bridges	23
2.8	A multigraph representing the Königsberg bridges	24
2.9	Which of these graphs contain Euler walks?	26
2.10	Which of these graphs contain Euler walks?	26
2.11	A graph with no Hamilton cycle	28
2.12	Steps in proving there is no Hamilton cycle	29
2.13	A bipartite graph with no Hamilton cycle	29
2.14	Graphs for Exercise 2.4.1	30
2.15	Graphs for Exercise 2.4.2	30