

# Contents

CHAPTER 1	Fundamental Concepts of Group Theory	1
1.1.	Binary Operations, Semigroups, and Groups	1
1.2.	Examples of Groups	4
1.3.	Subgroups and Cosets	8
1.4.	Homomorphisms and Quotient Groups	17
1.5.	Endomorphisms and Automorphisms	25
1.6.	Permutation Groups and Group Actions	31
CHAPTER 2	Free Groups and Presentations	44
2.1.	Free Groups	44
2.2.	Presentations of Groups	50
2.3.	Varieties of Groups	56
CHAPTER 3	Decompositions of a Group	63
3.1.	Series and Composition Series	63
3.2.	Some Simple Groups	71
3.3.	Direct Decompositions	80

<b>CHAPTER 4</b>	
<b>Abelian Groups</b>	93
4.1. Torsion Groups and Divisible Groups	93
4.2. Direct Sums of Cyclic and Quasicyclic Groups	98
4.3. Pure Subgroups and $p$ -Groups	106
4.4. Torsion-Free Groups	114
<b>CHAPTER 5</b>	
<b>Soluble and Nilpotent Groups</b>	121
5.1. Abelian and Central Series	121
5.2. Nilpotent Groups	129
5.3. Groups of Prime-Power Order	139
5.4. Soluble Groups	147
<b>CHAPTER 6</b>	
<b>Free Groups and Free Products</b>	159
6.1. Further Properties of Free Groups	159
6.2. Free Products of Groups	167
6.3. Subgroups of Free Products	174
6.4. Generalized Free Products	184
<b>CHAPTER 7</b>	
<b>Finite Permutation Groups</b>	192
7.1. Multiple Transitivity	192
7.2. Primitive Permutation Groups	197
7.3. Classification of Sharply $k$ -Transitive Permutation Groups	203
7.4. The Mathieu Groups	208
<b>CHAPTER 8</b>	
<b>Representations of Groups</b>	213
8.1. Representations and Modules	213
8.2. Structure of the Group Algebra	223
8.3. Characters	226
8.4. Tensor Products and Representations	235
8.5. Applications to Finite Groups	246
<b>CHAPTER 9</b>	
<b>Finite Soluble Groups</b>	252
9.1. Hall $\pi$ -Subgroups	252
9.2. Sylow Systems and System Normalizers	261
9.3. $p$ -Soluble Groups	269
9.4. Supersoluble Groups	274
9.5. Formations	277

<b>CHAPTER 10</b>	
<b>The Transfer and Its Applications</b>	285
10.1. The Transfer Homomorphism	285
10.2. Grün's Theorems	292
10.3. Frobenius's Criterion for $p$ -Nilpotence	295
10.4. Thompson's Criterion for $p$ -Nilpotence	298
10.5. Fixed-Point-Free Automorphisms	305
<b>CHAPTER 11</b>	
<b>The Theory of Group Extensions</b>	310
11.1. Group Extensions and Covering Groups	310
11.2. Homology Groups and Cohomology Groups	326
11.3. The Gruenberg Resolution	333
11.4. Group-Theoretic Interpretations of the (Co)homology Groups	341
<b>CHAPTER 12</b>	
<b>Generalizations of Nilpotent and Soluble Groups</b>	356
12.1. Locally Nilpotent Groups	356
12.2. Some Special Types of Locally Nilpotent Groups	363
12.3. Engel Elements and Engel Groups	369
12.4. Classes of Groups Defined by General Series	376
12.5. Locally Soluble Groups	381
<b>CHAPTER 13</b>	
<b>Subnormal Subgroups</b>	385
13.1. Joins and Intersections of Subnormal Subgroups	385
13.2. Permutability and Subnormality	393
13.3. The Minimal Condition on Subnormal Subgroups	396
13.4. Groups in Which Normality Is a Transitive Relation	402
13.5. Automorphism Towers and Complete Groups	408
<b>CHAPTER 14</b>	
<b>Finiteness Properties</b>	416
14.1. Finitely Generated Groups and Finitely Presented Groups	416
14.2. Torsion Groups and the Burnside Problems	422
14.3. Locally Finite Groups	429
14.4. 2-Groups with the Maximal or Minimal Condition	437
14.5. Finiteness Properties of Conjugates and Commutators	439
<b>CHAPTER 15</b>	
<b>Infinite Soluble Groups</b>	450
15.1. Soluble Linear Groups	450
15.2. Soluble Groups with Finiteness Conditions on Abelian Subgroups	455

15.3. Finitely Generated Soluble Groups and the Maximal Condition on Normal Subgroups	461
15.4. Finitely Generated Soluble Groups and Residual Finiteness	470
15.5. Finitely Generated Soluble Groups and Their Frattini Subgroups	474
<b>Bibliography</b>	479
<b>Index</b>	491
CHAPTER 5	
Soluble and Nilpotent Groups	
16.1. Abelian and Central Series	493
16.2. Nilpotent Groups	494
16.3. Groups of Prime-Power Order	495
16.4. Soluble Groups	497
16.5. The Orlon-Popovitz Representation of Finite Cyclic Groups	498
CHAPTER 6	
Free Groups and Free Products	
17.1. Further Properties of Free Groups and Subgroups	511
17.2. Free Products of Groups	512
17.3. Subgroups of Free Products	513
17.4. Centralized Free Products	514
17.5. Groups of Cyclic Subgroups	515
CHAPTER 7	
Finite Permutation Groups	
18.1. Multiple Transitivity	521
18.2. Primitive Permutation Groups	522
18.3. Classification of Sharply Transitive Groups	523
18.4. The Mathieu Groups	525
18.5. The Janko Groups	526
18.6. The PSL Groups	527
CHAPTER 8	
Representations of Groups	
19.1. Representations and Modules	535
19.2. Structure of the Group Algebra	536
19.3. Characters	537
19.4. Primary Classification Groups and Linear Representations	538
19.5. Applications to Finite Groups	539
19.6. Finite Linear Groups	540
19.7. Groups with the Maximal Condition	541
19.8. Finite Groups of Odd Order and Commutation	542
CHAPTER 9	
Solvable Groups	
20.1. Hall Subgroups	551
20.2. Sylow Systems and Sylow Normalizers	552
20.3. Sylow Groups	553
20.4. Sylow Subgroups	554
20.5. Solvable Groups with Finite Commutation Condition	555
CHAPTER 10	
Nilpotent Groups	
21.1. Nilpotent Groups	563
21.2. Solvable Groups	564
21.3. Nilpotent Groups and Commutativity	565
21.4. Nilpotent Groups and Nilpotent Quotients	566
21.5. Nilpotent Groups and Cohomology	567
21.6. Nilpotent Groups and Cohomology	568
21.7. Nilpotent Groups and Cohomology	569
21.8. Nilpotent Groups and Cohomology	570
21.9. Nilpotent Groups and Cohomology	571
21.10. Nilpotent Groups and Cohomology	572
21.11. Nilpotent Groups and Cohomology	573
21.12. Nilpotent Groups and Cohomology	574
21.13. Nilpotent Groups and Cohomology	575
21.14. Nilpotent Groups and Cohomology	576
21.15. Nilpotent Groups and Cohomology	577
21.16. Nilpotent Groups and Cohomology	578
21.17. Nilpotent Groups and Cohomology	579
21.18. Nilpotent Groups and Cohomology	580
21.19. Nilpotent Groups and Cohomology	581
21.20. Nilpotent Groups and Cohomology	582
21.21. Nilpotent Groups and Cohomology	583
21.22. Nilpotent Groups and Cohomology	584
21.23. Nilpotent Groups and Cohomology	585
21.24. Nilpotent Groups and Cohomology	586
21.25. Nilpotent Groups and Cohomology	587
21.26. Nilpotent Groups and Cohomology	588
21.27. Nilpotent Groups and Cohomology	589
21.28. Nilpotent Groups and Cohomology	590
21.29. Nilpotent Groups and Cohomology	591
21.30. Nilpotent Groups and Cohomology	592
21.31. Nilpotent Groups and Cohomology	593
21.32. Nilpotent Groups and Cohomology	594
21.33. Nilpotent Groups and Cohomology	595
21.34. Nilpotent Groups and Cohomology	596
21.35. Nilpotent Groups and Cohomology	597
21.36. Nilpotent Groups and Cohomology	598
21.37. Nilpotent Groups and Cohomology	599
21.38. Nilpotent Groups and Cohomology	600
21.39. Nilpotent Groups and Cohomology	601
21.40. Nilpotent Groups and Cohomology	602
21.41. Nilpotent Groups and Cohomology	603
21.42. Nilpotent Groups and Cohomology	604
21.43. Nilpotent Groups and Cohomology	605
21.44. Nilpotent Groups and Cohomology	606
21.45. Nilpotent Groups and Cohomology	607
21.46. Nilpotent Groups and Cohomology	608
21.47. Nilpotent Groups and Cohomology	609
21.48. Nilpotent Groups and Cohomology	610
21.49. Nilpotent Groups and Cohomology	611
21.50. Nilpotent Groups and Cohomology	612
21.51. Nilpotent Groups and Cohomology	613
21.52. Nilpotent Groups and Cohomology	614
21.53. Nilpotent Groups and Cohomology	615
21.54. Nilpotent Groups and Cohomology	616
21.55. Nilpotent Groups and Cohomology	617
21.56. Nilpotent Groups and Cohomology	618
21.57. Nilpotent Groups and Cohomology	619
21.58. Nilpotent Groups and Cohomology	620
21.59. Nilpotent Groups and Cohomology	621
21.60. Nilpotent Groups and Cohomology	622
21.61. Nilpotent Groups and Cohomology	623
21.62. Nilpotent Groups and Cohomology	624
21.63. Nilpotent Groups and Cohomology	625
21.64. Nilpotent Groups and Cohomology	626
21.65. Nilpotent Groups and Cohomology	627
21.66. Nilpotent Groups and Cohomology	628
21.67. Nilpotent Groups and Cohomology	629
21.68. Nilpotent Groups and Cohomology	630
21.69. Nilpotent Groups and Cohomology	631
21.70. Nilpotent Groups and Cohomology	632
21.71. Nilpotent Groups and Cohomology	633
21.72. Nilpotent Groups and Cohomology	634
21.73. Nilpotent Groups and Cohomology	635
21.74. Nilpotent Groups and Cohomology	636
21.75. Nilpotent Groups and Cohomology	637
21.76. Nilpotent Groups and Cohomology	638
21.77. Nilpotent Groups and Cohomology	639
21.78. Nilpotent Groups and Cohomology	640
21.79. Nilpotent Groups and Cohomology	641
21.80. Nilpotent Groups and Cohomology	642
21.81. Nilpotent Groups and Cohomology	643
21.82. Nilpotent Groups and Cohomology	644
21.83. Nilpotent Groups and Cohomology	645
21.84. Nilpotent Groups and Cohomology	646
21.85. Nilpotent Groups and Cohomology	647
21.86. Nilpotent Groups and Cohomology	648
21.87. Nilpotent Groups and Cohomology	649
21.88. Nilpotent Groups and Cohomology	650
21.89. Nilpotent Groups and Cohomology	651
21.90. Nilpotent Groups and Cohomology	652
21.91. Nilpotent Groups and Cohomology	653
21.92. Nilpotent Groups and Cohomology	654
21.93. Nilpotent Groups and Cohomology	655
21.94. Nilpotent Groups and Cohomology	656
21.95. Nilpotent Groups and Cohomology	657
21.96. Nilpotent Groups and Cohomology	658
21.97. Nilpotent Groups and Cohomology	659
21.98. Nilpotent Groups and Cohomology	660
21.99. Nilpotent Groups and Cohomology	661
21.100. Nilpotent Groups and Cohomology	662
21.101. Nilpotent Groups and Cohomology	663
21.102. Nilpotent Groups and Cohomology	664
21.103. Nilpotent Groups and Cohomology	665
21.104. Nilpotent Groups and Cohomology	666
21.105. Nilpotent Groups and Cohomology	667
21.106. Nilpotent Groups and Cohomology	668
21.107. Nilpotent Groups and Cohomology	669
21.108. Nilpotent Groups and Cohomology	670
21.109. Nilpotent Groups and Cohomology	671
21.110. Nilpotent Groups and Cohomology	672
21.111. Nilpotent Groups and Cohomology	673
21.112. Nilpotent Groups and Cohomology	674
21.113. Nilpotent Groups and Cohomology	675
21.114. Nilpotent Groups and Cohomology	676
21.115. Nilpotent Groups and Cohomology	677
21.116. Nilpotent Groups and Cohomology	678
21.117. Nilpotent Groups and Cohomology	679
21.118. Nilpotent Groups and Cohomology	680
21.119. Nilpotent Groups and Cohomology	681
21.120. Nilpotent Groups and Cohomology	682
21.121. Nilpotent Groups and Cohomology	683
21.122. Nilpotent Groups and Cohomology	684
21.123. Nilpotent Groups and Cohomology	685
21.124. Nilpotent Groups and Cohomology	686
21.125. Nilpotent Groups and Cohomology	687
21.126. Nilpotent Groups and Cohomology	688
21.127. Nilpotent Groups and Cohomology	689
21.128. Nilpotent Groups and Cohomology	690
21.129. Nilpotent Groups and Cohomology	691
21.130. Nilpotent Groups and Cohomology	692
21.131. Nilpotent Groups and Cohomology	693
21.132. Nilpotent Groups and Cohomology	694
21.133. Nilpotent Groups and Cohomology	695
21.134. Nilpotent Groups and Cohomology	696
21.135. Nilpotent Groups and Cohomology	697
21.136. Nilpotent Groups and Cohomology	698
21.137. Nilpotent Groups and Cohomology	699
21.138. Nilpotent Groups and Cohomology	700
21.139. Nilpotent Groups and Cohomology	701
21.140. Nilpotent Groups and Cohomology	702
21.141. Nilpotent Groups and Cohomology	703
21.142. Nilpotent Groups and Cohomology	704
21.143. Nilpotent Groups and Cohomology	705
21.144. Nilpotent Groups and Cohomology	706
21.145. Nilpotent Groups and Cohomology	707
21.146. Nilpotent Groups and Cohomology	708
21.147. Nilpotent Groups and Cohomology	709
21.148. Nilpotent Groups and Cohomology	710
21.149. Nilpotent Groups and Cohomology	711
21.150. Nilpotent Groups and Cohomology	712
21.151. Nilpotent Groups and Cohomology	713
21.152. Nilpotent Groups and Cohomology	714
21.153. Nilpotent Groups and Cohomology	715
21.154. Nilpotent Groups and Cohomology	716
21.155. Nilpotent Groups and Cohomology	717
21.156. Nilpotent Groups and Cohomology	718
21.157. Nilpotent Groups and Cohomology	719
21.158. Nilpotent Groups and Cohomology	720
21.159. Nilpotent Groups and Cohomology	721
21.160. Nilpotent Groups and Cohomology	722
21.161. Nilpotent Groups and Cohomology	723
21.162. Nilpotent Groups and Cohomology	724
21.163. Nilpotent Groups and Cohomology	725
21.164. Nilpotent Groups and Cohomology	726
21.165. Nilpotent Groups and Cohomology	727
21.166. Nilpotent Groups and Cohomology	728
21.167. Nilpotent Groups and Cohomology	729
21.168. Nilpotent Groups and Cohomology	730
21.169. Nilpotent Groups and Cohomology	731
21.170. Nilpotent Groups and Cohomology	732
21.171. Nilpotent Groups and Cohomology	733
21.172. Nilpotent Groups and Cohomology	734
21.173. Nilpotent Groups and Cohomology	735
21.174. Nilpotent Groups and Cohomology	736
21.175. Nilpotent Groups and Cohomology	737
21.176. Nilpotent Groups and Cohomology	738
21.177. Nilpotent Groups and Cohomology	739
21.178. Nilpotent Groups and Cohomology	740
21.179. Nilpotent Groups and Cohomology	741
21.180. Nilpotent Groups and Cohomology	742
21.181. Nilpotent Groups and Cohomology	743
21.182. Nilpotent Groups and Cohomology	744
21.183. Nilpotent Groups and Cohomology	745
21.184. Nilpotent Groups and Cohomology	746
21.185. Nilpotent Groups and Cohomology	747
21.186. Nilpotent Groups and Cohomology	748
21.187. Nilpotent Groups and Cohomology	749
21.188. Nilpotent Groups and Cohomology	750
21.189. Nilpotent Groups and Cohomology	751
21.190. Nilpotent Groups and Cohomology	752
21.191. Nilpotent Groups and Cohomology	753
21.192. Nilpotent Groups and Cohomology	754
21.193. Nilpotent Groups and Cohomology	755
21.194. Nilpotent Groups and Cohomology	756
21.195. Nilpotent Groups and Cohomology	757
21.196. Nilpotent Groups and Cohomology	758
21.197. Nilpotent Groups and Cohomology	759
21.198. Nilpotent Groups and Cohomology	760
21.199. Nilpotent Groups and Cohomology	761
21.200. Nilpotent Groups and Cohomology	762
21.201. Nilpotent Groups and Cohomology	763