

# Contents

**Preface** xv

**About the Author** xix

**About the Companion Website** xxi

## 1 **Introduction** 1

1.1 Reliability Defined 1

1.2 Performance, Cost, and Reliability 2

1.3 Quality, Reliability, and Safety Linkage 4

1.4 Quality, Reliability, and Safety Engineering Tasks 6

1.5 Preview 7

Bibliography 7

## 2 **Probability and Discrete Distributions** 9

2.1 Introduction 9

2.2 Probability Concepts 9

Relative Frequency 9

Classical 10

Subjective 10

*Sample space (S)* = set of all possible outcomes 10

*Outcome (e)* = an element of the sample space 10

*Event* = A subset of outcomes 11

Probability Axioms 11

More Than Two Events 17

Combinations and Permutations 21

2.3 Discrete Random Variables 23

Properties of Discrete Variables 23

The Binomial Distribution 26

The Poisson Distribution 30

Confidence Intervals 33

Motivation for Confidence Intervals 33

Introduction to Confidence Intervals 35

Binomial Confidence Intervals 37

Cumulative Sums of the Poisson Distribution (Thorndike Chart) 39

Bibliography 41

Advanced texts in Probability 41

Exercises 42

<b>3</b>	<b>The Exponential Distribution and Reliability Basics</b>	<b>47</b>
3.1	Introduction	47
3.2	Reliability Characterization	47
	Basic Definitions	48
	The Bathtub Curve	50
3.3	Constant Failure Rate Model	53
	The Exponential Distribution	54
	Demand Failures	55
	Time Determinations	57
3.4	Time-Dependent Failure Rates	61
3.5	Component Failures and Failure Modes	63
	Failure Mode Rates	63
	Component Counts	64
3.6	Replacements	67
3.7	Redundancy	71
	Active and Standby Redundancy	72
	Active Parallel	72
	Standby Parallel	73
	Constant Failure Rate Models	73
3.8	Redundancy Limitations	75
	Common-Mode Failures	76
	Load Sharing	77
	Switching and Standby Failures	79
	Cold, Warm, and Hot Standby	80
3.9	Multiply Redundant Systems	81
	1/N Active Redundancy	81
	1/N Standby Redundancy	83
	$m/N$ Active Redundancy	84
3.10	Redundancy Allocation	86
	High- and Low-level Redundancy	88
	Fail Safe and Fail to Danger	90
	Voting Systems	92
3.11	Redundancy in Complex Configurations	94
	Series-Parallel Configurations	94
	Linked Configurations	96
	Bibliography	98
	Exercises	98
	Redundancy	103
<b>4</b>	<b>Continuous Distributions – Part 1 Normal and Related Continuous Distributions</b>	<b>109</b>
4.1	Introduction	109
4.2	Properties of Continuous Random Variables	109
	Probability Distribution Functions	110
	Characteristics of a Probability Distribution	112
	Sample Statistics	114
	Transformations of Variables	115
4.3	Empirical Cumulative Distribution Function (Empirical CDF)	117
4.4	Uniform Distribution	120

4.5	Normal and Related Distributions	122
	The Normal Distribution	123
	Normal Distribution..... Cautions and Warnings!!	126
	Central Limit Theorem	127
	Central Limit Theorem in Practice	128
	The Lognormal Distribution	128
	Log Normal Distribution from a Physics of Failure Perspective	134
4.6	Confidence Intervals	135
	Point and Interval Estimates	135
	Estimate of the Mean	139
4.7	Normal and Lognormal Parameters	140
	Bibliography	142
	Exercises	143
5	<b>Continuous Distributions – Part 2 Weibull and Extreme Value Distributions</b>	149
5.1	Introduction	149
	The “Weakest Link” Theory from a Physics-of-Failure Point of View	149
	Uses of Weibull and Extreme Value Distributions	150
	Other Considerations	151
	Age Parameters and Sample Sizes	151
	Engineering Changes, Maintenance Plan Evaluation, and Risk Prediction	152
	Weibulls with Cusps or Curves	152
	System Weibulls	153
	No Failure Weibulls	154
	Small Sample Weibulls	154
	Summary	154
5.2	Statistics of the Weibull Distribution	154
	Weibull “Mathematics”	154
	The Weibull Probability Plot	158
	Probability Plotting Points – Median Ranks	160
	How to Do a “Weibull Analysis”	161
	Weibull Plots and Their Estimates of $\beta$ , $\eta$	163
	The Three-Parameter Weibull Did Not Work, What Are My Choices?	167
	The Data has a “Dogleg” Bend or Cusp When Plotted on Weibull Paper	167
	Steep Weibull Slopes ( $\beta$ s) May Hide Problems	171
	Low-Time Failures and Close Serial numbers – Batch Problems	172
	Maximum-Likelihood Estimates of $\beta$ and $\eta$	172
	Weibayes Analysis	176
	Weibayes Background (You Do Not Necessarily Have Any Failure Times)	177
	Weibull Analysis with Failures Only and Unknown Times on the Unfailed Population	180
	Shifting Weibull Procedure	180
	Confidence Bounds and the Weibull Distribution	181
	Arbitrary Censored Data – Left-Censored, Right-Censored, and Interval Data	184
	The Weibull Distribution in a System of Independent Failure Modes	188
5.3	Extreme Value Distributions	189
5.4	Introduction to Risk Analysis	197
	Risk Analysis “Mathematics”	197
	Bibliography	203

	Exercises	205
	Supplement 1: Weibull Derived from Weakest Link Theory	219
<b>6</b>	<b>Reliability Testing</b>	221
6.1	Introduction	221
6.2	Attribute Testing (Binomial Testing)	223
	The Classical Success Run	224
	Zero-Failure Attribute Tests	224
	Non-Zero-Failure Attribute Tests	225
6.3	Constant Failure Rate Estimates	228
	Censoring on the Right	228
	MTTF Estimates	230
	Confidence Intervals	232
6.4	Weibull Substantiation and Reliability Testing	234
	Zero-Failure Test Plans for Substantiation Testing	235
	Weibull Zero-Failure Test Plans for Reliability Testing	237
	Reexpression of a Reliability Goal to Determine $\eta$	239
	Designing the Test Plan	239
	Test Units with Censored Times (due to Julius Wang, Fiat-Chrysler)	241
	Total Test Time	242
	Why Not Simply Test to Failure?	243
6.5	How to Reduce Test Time	243
	Run (Simultaneously) More Test Samples Than You Intend to Fail	243
	Sudden Death Testing	245
	Sequential Testing	247
6.6	Normal and Lognormal Reliability Testing	255
6.7	Accelerated Life Testing	262
	Compressed-Time Testing	262
	Advanced-Stress Testing – Linear and Acceleration Models	265
	Linear Model Stress Testing	266
	Advanced-Stress Testing – Acceleration Models	270
	The Arrhenius Model	270
	The Inverse Power Law Model	275
	Other Acceleration Models	280
6.8	Reliability-Enhancement Procedures	282
	Reliability Growth Modeling and Testing	282
	Calculation of Reliability Growth Parameters	287
	Goodness-of-Fit Tests for Reliability Growth Models	288
	For Time-Terminated Testing	288
	For Failure-Terminated Testing	289
	For Grouped Data	289
	Environmental Stress Screening	299
	What “Screens” are used for ESS?	302
	Thermal Cycling	302
	Random Vibration	303
	Other Screens	303

Highly Accelerated Life Tests	304
Highly Accelerated-Stress Screening	305
Bibliography	305
Exercises	306
Supplement 1: Tables for Weibull Zero-failure Substantiation testing	315
Supplement 2: Tables For Weibull Zero-failure Substantiation testing using (t/Eta)	319
Supplement 3: Critical Values for Cramer-Von Mises Goodness-of-Fit Test	323
Supplement 4: Other Reliability Growth Models that have been Proposed and Studied (see AFWAL-TR-84-2024 for details)	323
(a) Deterministic Models	323
(b) Poisson Process Models	324
(c) Markov Processes/Time Series Models	325
Supplement 5: Chi-Square Table	326

## 7 Failure Modes and Effects Analysis – Design and Process 327

7.1	Introduction	327
7.2	Functional FMEA	328
7.3	Design FMEA	332
	Design FMEA Procedure	332
7.4	Process FMEA (PFMEA)	339
7.5	FMEA Summary	349
	Bibliography	350
	Exercises	350
	Supplement 1: Shortcut Tables for Stalled FMEA Teams	359
	Supplement 2: Future Changes in FMEA Approaches	360
	Supplement 3: DFMEA and PFMEA Forms	360

## 8 Loads, Capacity, and Reliability 361

8.1	Introduction	361
8.2	Reliability with a Single Loading	362
	Load Application	363
	Definitions	364
8.3	Reliability and Safety Factors	368
	Normal Distributions	368
	Lognormal Distributions	373
	Combined Distributions	374
8.4	Repetitive Loading	376
	Loading Variability	376
	Variable Capacity	380
8.5	The Bathtub Curve – Reconsidered	382
	Single Failure Modes	383
	Combined Failure Modes	385
	Bibliography	387
	Exercises	388
	Supplement 1: The Dirac Delta Distribution	392

<b>9</b>	<b>Maintained Systems</b>	395
9.1	Introduction	395
9.2	Preventive Maintenance	396
	Idealized Maintenance	396
	Imperfect Maintenance	401
	Redundant Components	403
9.3	Corrective Maintenance	403
	Availability	404
	Maintainability	405
9.4	Repair: Revealed Failures	407
	Constant Repair Rates	407
	Constant Repair Times	410
9.5	Testing and Repair: Unrevealed Failures	411
	Idealized Periodic Tests	411
	Real Periodic Tests	413
9.6	System Availability	415
	Revealed Failures	416
	Unrevealed Failures	418
	Simultaneous Testing	419
	Staggered Testing	420
	Bibliography	422
	Exercises	422
<b>10</b>	<b>Failure Interactions</b>	427
10.1	Introduction	427
10.2	Markov Analysis	427
	Two Independent Components	429
	Load-Sharing Systems	432
10.3	Reliability With Standby Systems	434
	Idealized System	434
	Failures in the Standby State	437
	Switching Failures	439
	Primary System Repair	442
10.4	Multicomponent Systems	444
	Multicomponent Markov Formulations	444
	Combinations of Subsystems	448
10.5	Availability	449
	Standby Redundancy	449
	Shared Repair Crews	453
	Markov Availability – Advantages and Disadvantages	457
	The Advantages of Markov Availability Analysis	457
	The Disadvantages of Markov Availability Analysis	457
	Bibliography	457
	Exercises	457
<b>11</b>	<b>System Safety Analysis</b>	463
11.1	Introduction	463
11.2	Product and Equipment Hazards	464
11.3	Human Error	466

11.4	Routine Operations 468
	Emergency Operations 470
	Methods of Analysis 471
	Failure Modes, Effects, and Criticality Analysis (FMECA) 472
	Criticality 472
	Event Trees 478
11.5	Fault Trees 480
	Fault-Tree Construction 482
	Nomenclature 483
	Fault Classification 486
	Primary, Secondary, and Command Faults 486
	Passive and Active Faults 486
	Fault Tree Examples 487
	Direct Evaluation of Fault Trees 494
	Qualitative Evaluation 494
	Top Down 495
	Bottom Up 495
	Logical Reduction 496
	Quantitative Evaluation 496
	Probability Relationships 497
	Primary-Failure Data 498
	Fault-Tree Evaluation by Cut Sets 499
	Qualitative Analysis 499
	Minimum Cut-Set Formulation 499
	Cut-Set Determination 501
	Cut-Set Interpretations 502
	Quantitative Analysis 503
	Top-Event Probability 503
	Importance 505
	Uncertainty 505
11.6	Reliability/Safety Risk Analysis 505
	Conclusion: Assuming Worst Case can be Misleading 508
	Another Approach: Monte Carlo Simulation 508
	Bibliography 515
	FMEA/FMECA 515
	Exercises 516

## **Appendix A: Useful Mathematical Relationships 521**

A.1	Integrals 521
	Definite Integrals 521
	Integration by Parts 521
	Derivative of an Integral 521
A.2	Expansions 522
	Integer Series 522
	Binomial Expansion 522
	Geometric Progression 522
	Infinite Series 522
A.3	Solution of First-order Linear Differential Equation 523

<b>Appendix B: Binomial Failure Probability Charts</b>	525
<b>Appendix C: <math>\Phi(z)</math>: Standard Normal CDF</b>	529
<b>Appendix D: Nonparametric Methods and Probability Plotting</b>	
D.1 Introduction	533
D.2 Nonparametric Methods for Probability Plotting	533
Boxplots and Histograms	533
Boxplot	533
Histogram	535
Rank Statistics	536
D.3 Parametric Methods	537
Weibull Distribution Plotting	540
Extreme-Value Distribution Plotting	543
Lognormal Distribution Plotting	545
D.4 Goodness of Fit	547
Bibliography	555
<b>3rd Ed Answers to Odd – Numbered Exercises</b>	557
<b>Index</b>	607