

# Table of Contents

<b>1. Introduction</b>	1
1.1 Curves and Surfaces in Space	3
1.1.1 Mathematical Description	3
1.1.2 Rigid Body Motion of Model Features in Space	5
1.1.3 Model Hierarchy	6
1.2 Curve and Surface Fitting	9
1.2.1 Applications of Curve and Surface Fitting	9
1.2.2 Algebraic Fitting Vs. Geometric Fitting	9
1.2.3 State-of-the-Art Orthogonal Distance Fitting	13
1.2.4 ISO 10360-6 and Requirements of CMM Software Tools	14
<b>2. Least-Squares Orthogonal Distance Fitting</b>	17
2.1 Moment Method for Line and Plane Fitting	17
2.1.1 Line Fitting	17
2.1.2 Plane Fitting	20
2.1.3 Relationship Between Line and Plane Fitting	21
2.2 Generalized Orthogonal Distance Fitting	22
2.2.1 Problem Definition	22
2.2.2 Point-to-Point Matching	24
2.2.3 Curve and Surface Fitting	24
2.2.4 Template Matching	26
2.3 Orthogonal Distance Fitting Algorithms	27
2.3.1 Distance-Based Algorithm	27
2.3.2 Coordinate-Based Algorithm	28
2.3.3 Model Fitting with Parameter Constraints	29
2.3.4 Parameter Test	30
2.3.5 Application to Circle and Sphere Fitting	31
<b>3. Orthogonal Distance Fitting of Implicit Curves and Surfaces</b>	35
3.1 Minimum Distance Point	36
3.1.1 Generalized Newton Method	37
3.1.2 Method of Lagrangian Multipliers	39
3.1.3 Verification of the Minimum Distance Point	40
3.1.4 Acceleration of Finding the Minimum Distance Point	41

3.2	Orthogonal Distance Fitting .....	42
3.2.1	Distance-Based Algorithm .....	42
3.2.2	Coordinate-Based Algorithm .....	44
3.2.3	Comparison of the Two Algorithms .....	46
3.3	Fitting Examples .....	47
3.3.1	Superellipse Fitting .....	47
3.3.2	Cone Fitting .....	48
3.3.3	Torus Fitting .....	50
3.3.4	Superellipsoid Fitting .....	51
<b>4.</b>	<b>Orthogonal Distance Fitting of Parametric Curves and Surfaces .....</b>	<b>55</b>
4.1	Minimum Distance Point .....	56
4.1.1	Newton Method .....	57
4.1.2	Levenberg-Marquardt Algorithm .....	58
4.1.3	Initial Values .....	60
4.1.4	Acceleration of Finding the Minimum Distance Point .....	62
4.2	Orthogonal Distance Fitting .....	63
4.2.1	Algorithm I (ETH) .....	63
4.2.2	Algorithm II (NPL, FhG) .....	65
4.2.3	Algorithm III (FhG) .....	67
4.2.4	Comparison of the Three Algorithms .....	69
4.3	Fitting Examples .....	70
4.3.1	Helix Fitting .....	70
4.3.2	Ellipsoid Fitting .....	72
<b>5.</b>	<b>Object Reconstruction from Unordered Point Cloud .....</b>	<b>75</b>
5.1	Applications of Object Reconstruction .....	76
5.2	Semi-automatic Object Recognition .....	77
5.2.1	Segmentation, Outlier Elimination, and Model Fitting .....	78
5.2.2	Domain Volume for Measurement Points .....	79
5.3	Experimental Results with Real 3-D Measurement Points .....	80
5.3.1	3-D Point Cloud from Stripe Projection Method .....	80
5.3.2	3-D Point Cloud from Laser Radar .....	83
<b>6.</b>	<b>Conclusions .....</b>	<b>85</b>
6.1	Summary .....	85
6.2	Future Work .....	87
	<b>References .....</b>	<b>93</b>
	<b>Index .....</b>	<b>97</b>
<b>A.</b>	<b>Implementation Examples .....</b>	<b>101</b>
A.1	Implicit 2-D Ellipse (Chap. 3) .....	101
A.2	Parametric 3-D Ellipse (Chap. 4) .....	103

**B. CMM Software Tools Fulfilling ISO 10360-6** ..... 107

  B.1 Curves and Surfaces Defined in ISO 10360-6 ..... 107

    B.1.1 Competent Parameterization ..... 107

    B.1.2 Role of the Mass Center ..... 110

    B.1.3 Rotation Matrix ..... 111

    B.1.4 Parameter Range ..... 114

  B.2 Minimum Distance Point and FHG/XHG Matrix ..... 115

    B.2.1 2-D Line ..... 116

    B.2.2 3-D Line ..... 116

    B.2.3 Plane ..... 117

    B.2.4 2-D Circle ..... 117

    B.2.5 3-D Circle ..... 118

    B.2.6 Sphere ..... 118

    B.2.7 Cylinder ..... 119

    B.2.8 Cone ..... 119

    B.2.9 Torus ..... 120

**C. FHG Matrix of Superellipse and Superellipsoid** ..... 123

  C.1 Superellipse ..... 123

  C.2 Superellipsoid ..... 124

    Iterative search for the minimum distance point ..... 38

    Verification of the minimum distance point ..... 41

    Minimum distance points on the updated implicit feature ..... 42

    Information flow for orthogonal distance fitting ..... 43

    Orthogonal distance fitting to the points in Table 3.1 ..... 48

    Orthogonal distance fitting to the points in Table 3.3 ..... 49

    Orthogonal distance fitting to the points in Table 3.5 ..... 51

    Orthogonal distance fitting to the points in Table 3.7 ..... 52

  4.1 Parametric features and the minimum distance point ..... 56

  4.2 Search for the local minimum point  $u'$  of a function  $D(u)$  ..... 58

  4.3 Determination of  $\lambda$  for the Levenberg-Marquardt algorithm ..... 59

  4.4 Minimum distance point on the updated parametric feature ..... 62

  4.5 Information flow with Algorithm III (FHG) for parametric features ..... 68

  4.6 Orthogonal distance fitting to the points in Table 4.3 ..... 71

  4.7 Ellipsoid fit to 10–100 points ..... 72

  4.8 Convergence and comparing cost of the three Algorithms I–III ..... 72

  5.1 Object recognition scheme ‘Click & Clear’ in a point cloud ..... 77

  5.2 Example of the domain area for an ellipse ..... 79

  5.3 Sphere-like object surface ..... 81

  5.4 Results of model fitting to the point cloud from Fig. 5.3 ..... 81

  5.5 Point cloud taken by laser radar ..... 83

  5.6 Results of object recognition in the point cloud of Fig. 5.5 ..... 84