I I The Perpendicusar Projection Theorem enoughter of and Contents Implact of a Linear Operator on a Vector. Inequalities Concerning the Doll Product quillow rating bru mus

The Cross Product and the Lox Product of Vectors Trigonometric Formula for the Length of the Cross Product

Formal Rules of the Cross Product

V

XVI

xvii

4

5

9

14

16

17

17

20

22

23

26

26

31

32

34

34

35

XI

Preface

The Greek Alphabet List of Applications

Introductory Remarks Concerning Logic 0

The Notion of a Function **Injections and Surjections** Sufficient Condition. Necessary Condition The Principles of Negation The Jacobi Identity. The Concept of The Axiom of Choice Geometric Interpretation of the Jacobi Identi Exercises

The Sum and the Scalar Multiple of Vectors Formal Rules of the Sum

Formal Rules of the Scalar Multiple Formal Properties of the Negative of a Vector Location Vectors. Coordinates. Direction Numbers Linear Combinations Genergal, Engastion, et Me, Rians, H Applications Normal Equation of the Plane **Coordinate Formulas** Exercises

The Dot Product of Vectors

Trigonometric Formula for the Dot Product Application

	The Perpendicular Projection Theorem	36
	Formal Rules of the Dot Product	36
	Applications	38
	Coordinate Formulas	45
	Direction Cosines	46
	Orthonormal Bases	48
	Applications	48
	Inequalities Concerning the Dot Product	53
	Exercises	54
3	The Cross Product and the Box Product of Vectors	57
	Trigonometric Formula for the Length of the Cross Product	57
	Application	59
	The Cross Product As a Linear Operator	59
	Formal Rules of the Cross Product	60
	Application	61
	Coordinate Formulas for the Cross Product	62
	Applications	62
	Box Product of Three Vectors	65
	Coordinate Formula for the Box Product	67
	Applications	67
	Exercises	69
4		72
	The Jacobi Identity. The Concept of a Lie Algebra	72
	Geometric Interpretation of the Jacobi Identity	73
	The Double Cross Formula	74
	Applications	77
	The Lie Algebra of Antisymmetric Operators	83
	Exercises	85

5	Lines and Planes	87
	Coordinates in Three-Dimensional Space	87
	General Equation of the Plane	92
	Normal Equation of the Plane	96
	Applications	97
	Parametric and Symmetric Forms of a Line	98
	Applications	99
	Transversals: Applications	104
	Parametric Form of the Plane	106
	Exercises	107

6	Linear Operators	110
	Some Simple Examples of Linear Operators	110
	Concept of a Linear Operator	117
	Matrix of a Linear Operator	118
	Impact of a Linear Operator on a Vector	121
	Applications	122
	Sum and Scalar Multiple of Linear Operators	124
	Applications	125
	Exercises	132
7	The Product of Linear Operators	135
	Formal Rules of the Product of Linear Operators	135
	Applications	137
	The Matrix Product Rule	143
	Applications	145
	Exercises	151
8	Invertible Linear Operators	155
	Necessary and Sufficient Condition for Invertibility	155
	Formal Rules of the Inverse	157
	The General Linear Group	157
	Applications	158
	The Inverse Matrix Formula	160
	Second Method for Calculating the Inverse Matrix	163
	Applications	165
	Exercises	166
9	Kernel and Image: The Classification of Linear Maps	169
	Kernel and Image	169
	Applications	170
	The Law of Nullity. Rank	172
	Applications	174
	Classification of Linear Maps	176
	Necessary and Sufficient Conditions for the Invertibility of Linear Maps and Operators	177
	Standard Form of a Matrix. Calculation of the Rank	178
	Homogeneous Systems of Linear Equations	182
	Applications	186
	Nonhomogeneous Systems of Linear Equations	189
	Applications	196
	Exercises	201

.

xiii

.

10	Eigenvalues and Eigenvectors: The Classification of Linear	
	Operators	206
	Existence and Uniqueness of Eigenvectors	206
	Necessary and Sufficient Conditions for the Invertibility of	
	a Linear Operator	208
	Applications	208
	Finding the Eigenvectors if the Eigenvalues Are Known	214
	Applications	215
	Finding the Eigenvalues of a Linear Operator	217
	Application	225
	Classification of Linear Operators	229
	Applications	236
	Type of Linear Operators in \mathbb{R}^n	240
	Exercises	241
11	Symmetric, Antisymmetric, and Orthogonal Operators	248
	The Star-Dot Formula	248
	Applications	249
	Formal Properties of Transposition	050
	Symmetric Operators. The Principal Axis Theorem	251
	Applications	256
	Antisymmetric Operators. The Principal Twist Theorem	257
	Orthogonal Operators. The Orthogonal Group	260
	Applications	263
	Exercises	270
12	Rotations and Reflections: The Classification of Orthogonal	
	Operators	281
	Case of \mathbb{R}^2	281
	Application	
	Complex Numbers	
	Application	
	Case of \mathbb{R}^3	
	Applications	295
	Case of \mathbb{R}^n	
	Exercises	308
13	The Determinant and the Trace of a Linear Operator	
	Determinant of a Linear Operator	311
	Formal Properties of the Determinant	312
	Orientation in \mathbb{R}^n	314
		514

xiv

	Volume in \mathbb{R}^n	315
	Application	318
	Trace of a Linear Operator	319
	Formal Properties of the Trace	320
	Application	321
	Exercises	321
	LACICIDED	541
14	The Exponential Functor	325
	The Exponential of a Linear Operator	325
	Formal Properties of the Exponential	327
	Geometric Meaning of the Trace	328
	Applications	330
	Streamlines of Operators of \mathbb{R}^2 by Type	332
	Applications	335
	Lie Algebras of Linear Operators	339
	Conjugation	343
	The Concept of a Functor	345
	Changing the Metric	350
	Exercises	352
App	pendix 1 Numerical Methods of Linear Algebra:	
Det	erminants. Gaussian Elimination	355
	Definition of an $n \times n$ Determinant	355
	Properties of $n \times n$ Determinants	356
	Applications	361
	Cramer's Rule	365
	Method of Gaussian Elimination	368
App	pendix 2 The Field of Real Numbers	372
	Concept of a Complete Ordered Field	372
	Field of Rational Numbers	373
	Least Upper Bound. Greatest Lower Bound	374
	Irrational Numbers	375
	Algebraic Closure	376
~ .	17. Resident forms of the circle	
Solu	utions to Exercises	377
Ref	erences and Selected Readings	413
	L. Heroms formula	
Ind	ex	415

XV