

This book provides an overview of the myriad methods for applying dynamical systems techniques to PDEs and highlights the impact of PDE methods on dynamical systems. Also included are many nonlinear evolution equations, which have been benchmark models across the sciences, and examples and techniques to strengthen preparation for research.

PDE Dynamics: An Introduction is intended for senior undergraduate students, beginning graduate students, and researchers in applied mathematics, theoretical physics, and adjacent disciplines.

Structured as a textbook or seminar reference, it can be used in courses titled Dynamics of PDEs, PDEs 2, Dynamical Systems 2, Evolution Equations, or Infinite-Dimensional Dynamics.

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