

Contents

1. Signals and systems and their mathematical models	17
1.1 Introduction	17
1.2 Classification of signals	18
1.3 Basic operations with continuous-time signals	20
1.3.1 Shift in time	21
1.3.2 Inversion of time axis	21
1.3.3 Inversion of time axis with shift	21
1.3.4 Signal amplification, attenuation and inversion	22
1.3.5 Changing the time scale	22
1.3.6 Addition, subtraction, multiplication, and division of two signals	22
1.3.7 Convolution	23
1.3.8 Correlation	25
1.4 Basic continuous-time signals	26
1.5 Continuous-time systems	27
1.5.1 Model of an electrical system	28
1.5.2 Classification of systems	31
1.5.3 Linear time-invariant continuous-time systems	33
2. Periodic signals and their spectrum	34
2.1 Introduction	34
2.2 Definition of the Fourier series and its forms	35
2.3 Complex model of harmonic signals	36
2.4 Complex form of the Fourier series	37
2.5 Example of calculating coefficients of the Fourier series in complex form	38
2.6 Gibb's phenomenon	39
2.7 Properties of the Fourier series	40
2.7.1 Mean power of periodic signals	40
2.7.2 Spectrum of a sum of periodic signals	41
2.7.3 Spectrum of periodic signal multiplied by a constant	41
2.7.4 Spectrum of periodic signal shifted in time	41
2.7.5 Spectrum of periodic signal with changed time scale	43
2.8 Example of calculating coefficients of the Fourier series in the first form	44
2.9 Correlation and autocorrelation functions of periodic signals	47
2.10 Quasi-periodic signals	49
3. Fourier representation of aperiodic continuous-time signals	50
3.1 Introduction	50
3.2 Definition of the Fourier transform	50
3.3 Example of calculating the Fourier transform	52
3.4 Relationship between the Fourier series and the Fourier transform	52
3.5 Properties of the Fourier transform	57
3.5.1 Fourier transform of real signals	57
3.5.2 Linearity of the Fourier transform	58
3.5.3 Change of time axis sign	58
3.5.4 Shift in time	58

3.5.5	Change in time scale	58
3.5.6	Fourier transform of convolution	58
3.5.7	Parseval's theorem	59
3.6	Spectra of selected aperiodic signals.	59
3.6.1	Fourier transform of Dirac impulse	59
3.6.2	Fourier transform of unit step	60
3.6.3	Fourier transform of harmonic signal.	62
3.6.4	Fourier transform of rectangular window.	62
3.6.5	Time function for rectangular frequency window	63
3.6.6	Fourier transform of Gaussian impulse	63
3.6.7	Fourier transform of finite segment of harmonic signal.	64
3.6.8	Fourier transform of periodic sequence of Dirac impulses	65
4.	Continuous-time systems	67
4.1	Frequency response	67
4.2	Example of calculating frequency response.	68
4.3	Frequency response and the Laplace transform	70
4.4	Ideal transfer circuit.	70
4.5	Frequency filters	71
4.6	Procedure of frequency filter design.	72
4.6.1	Determining frequency filter requirements	72
4.6.2	Approximation of requirements using mathematical functions.	73
4.6.3	Choice of approximation function implementation	73
4.6.4	Verification of correct function of a frequency filter.	74
4.7	Complete solution, zero input response and zero state response	74
4.7.1	First-order electric circuit with DC source.	74
4.7.2	Second-order electric circuit with DC source	76
4.7.3	First-order electric circuit with harmonic source.	83
4.8	Non-linear system and its model	85
4.8.1	Model of non-linear systems	85
4.8.2	Receiver with indirect amplification - superheterodyne	86
5.	Sampling of continuous-time signals	89
5.1	Introduction	89
5.2	Ideal sampling	95
5.3	Recovery of continuous-time signal from its samples.	98
5.4	Sampling of band-limited signals	100
5.5	First-order sampling.	101
5.6	Second-order sampling.	103
5.7	Amplitude quantization	105
6.	Discrete-time signals	107
6.1	Introduction	107
6.2	Basic one-dimensional discrete signals	109
6.3	Basic two-dimensional discrete signals	114
6.4	Operations with one-dimensional discrete signals	118
6.4.1	Addition and multiplication of discrete signals	118
6.4.2	Modulo N operation.	118
6.4.3	Rectangular window sequence	118
6.5	Linear discrete convolution	118
6.6	Periodic discrete convolution	119
6.7	Fast discrete convolution	120
6.8	Circular discrete convolution, circular shift	121

7.	Fourier representation of discrete signals	124
7.1	Fourier transform of discrete signals	124
7.2	Discrete Fourier series	125
7.3	Spectrum of a finite section of a harmonic signal	127
7.4	Condition of correct interpretation of the discrete Fourier series	131
7.5	Example of calculating the discrete Fourier series	133
7.6	Properties of the discrete Fourier series	134
7.6.1	Linearity of the discrete Fourier series	134
7.6.2	Discrete Fourier series of a real sequence	135
7.6.3	Discrete Fourier series of a shifted sequence	135
7.6.4	Discrete Fourier series of a periodic convolution	135
7.6.5	Discrete Fourier series of the product of periodic discrete signals	136
7.7	Discrete Fourier transform	136
7.7.1	Definition of the discrete Fourier transform	136
7.7.2	Example of calculating the discrete Fourier transform	137
7.8	Properties of the discrete Fourier transform	138
7.8.1	Linearity of the discrete Fourier transform	138
7.8.2	Discrete Fourier transform of a real sequence	138
7.8.3	Discrete Fourier transform of a cyclically shifted sequence	138
7.8.4	Discrete Fourier transform of a circular (cyclic) convolution	138
7.9	Fast Fourier transform	138
7.10	Discrete cosine transform	143
7.10.1	Even and odd periodic sequences obtained from an aperiodic sequence	143
7.10.2	Definition of the one-dimensional discrete cosine transform DCT-II	144
7.10.3	Relation between DCT-II and DFT	144
7.10.4	Two-dimensional discrete cosine transform DCT-II	147
8.	$\mathcal{Z}$ transform and its properties	149
8.1	Definition of the one-sided $\mathcal{Z}$ transform	149
8.2	Properties of the one-sided $\mathcal{Z}$ transform	150
8.2.1	Linearity	150
8.2.2	Sequence delay (right-shifting)	152
8.2.3	Sequence advance (left-shifting)	152
8.2.4	Differentiation of the $\mathcal{Z}$ transform	152
8.2.5	Convolution of two discrete signals	153
8.2.6	Summation of n sequence values	153
8.2.7	Product with a real exponential sequence	153
8.2.8	Initial value theorem	154
8.2.9	$\mathcal{Z}$ transform of a real sequence	154
8.3	Inverse one-sided $\mathcal{Z}$ transform	154
8.3.1	Decomposition into partial fractions and application of a dictionary	154
8.3.2	Calculation of the inverse $\mathcal{Z}$ transform using the residue theorem	155
8.4	Connection between the $\mathcal{Z}$ transform and the discrete Fourier transform	156
8.4.1	Discrete signal is of a finite length, which is equal to the size of the DFT	157
8.4.2	Discrete signal is of a finite length that exceeds the number of values of the DFT	157
8.5	One-sided $\mathcal{Z}$ transform applied to solve difference equations	158
9.	One-dimensional discrete systems	165
9.1	Linear time-invariant discrete systems	165
9.2	System transfer function of the LTI discrete system	167
9.3	Stability and causality of the LTI discrete system	170
9.3.1	Stability of the LTI discrete system	171
9.3.2	Causality of the LTI discrete system	173

9.3.3	Condition of real coefficients of the system transfer function of an LTI discrete system . . .	173
9.4	Frequency response of one-dimensional LTI discrete systems	174
9.4.1	Definition of the frequency response.	174
9.4.2	Relation between the system transfer function and the frequency response	175
9.4.3	Example of calculating the frequency response	175
9.4.4	Filtering of a discrete signal of a very long length	179
9.4.5	Calculation of the frequency response of an LTI discrete system from its time response . .	180
9.5	Connection of partial sections of one-dimensional LTI discrete systems.	182
9.5.1	Connection in series (cascade)	182
9.5.2	Connection in parallel	182
9.5.3	Feedback connection	183
9.5.4	Example of connecting two partial first-order LTI discrete systems	184
10.	Analysis of one-dimensional discrete systems	190
10.1	Difference equations applied for description of LTI discrete systems	190
10.1.1	Difference equation solution by the method of difference equation analysis	191
10.1.2	Z transform applied to solve difference equations	193
10.2	State-space matrix representation of one-dimensional LTI discrete systems	194
10.3	Canonical structures of LTI discrete systems.	197
10.4	Computer analysis of one-dimensional LTI discrete systems	199
10.4.1	Matrix equations for computer analysis	199
10.4.2	Mason's gain rule for the signal flow graph transfer calculation	202
10.4.3	One-dimensional LTI discrete system realization check	202
10.5	One-dimensional LTI discrete systems with minimum, mixed and maximum phase	204
10.5.1	All-pass filter	206
10.5.2	Minimum-phase LTI discrete systems.	208
10.5.3	Compensation of properties of a discrete system with undesirable frequency response . .	209
10.6	Calculation of DFT spectrum by means of digital filtering	210
10.6.1	Goertzel algorithm	210
10.6.2	Example of frequency response calculation by the Goertzel algorithm	214
10.7	Autonomous discrete system and its application	216
10.7.1	Discrete harmonic signal generation	216
10.7.2	Example of using an autonomous discrete system	218
10.8	Adaptive discrete systems	220
10.8.1	Definition of the adaptive discrete system	220
10.8.2	Optimal Wiener filtering	221
10.8.3	Adaptive filtering by means of the LMS algorithm	223
11.	Two-dimensional systems with discrete time	224
11.1	Linear two-dimensional shift-invariant discrete systems.	224
11.2	Stability and causality of 2D discrete linear shift-invariant systems.	229
11.3	Frequency response of 2D discrete LSI systems.	230
11.4	Linear 2D difference equations with constant coefficients	231
11.5	Connecting partial sections of 2D discrete LSI systems	231
12.	Multirate digital signal processing	233
12.1	Down-sampling of discrete signals in integer number ratio	233
12.2	Up-sampling of discrete signals in integer number ratio	238
12.3	Down-sampling and up-sampling in rational number ratio	239
12.4	Optimization of arithmetic operations and sizes of lowpass filters with various numbers of sections	242

13. Band-limited signal representation	247
13.1 Continuous band-limited signals	247
13.1.1 Hilbert transform	248
13.1.2 Real band-limited signal calculated from the complex envelope	249
13.1.3 Calculation of the complex envelope from the real band-limited signal	251
13.1.4 Instantaneous amplitude, instantaneous frequency and instantaneous phase	253
13.2 Sampling of band-limited continuous-time signals	253
13.2.1 Conditions for band-limited signal sampling	253
13.2.2 Example of continuous band-limited signal sampling	254
13.2.3 Continuous band-limited signal sampling in quadrature modulator	254
13.3 Discrete-time band-limited signals	255
14. Digital filter banks	257
14.1 Analysis and synthesis filter banks	257
14.2 Uniform filter banks	257
14.2.1 Calculation of DFT spectrum by means of uniform digital filter bank	257
14.2.2 Polyphase discrete signal processing	263
14.3 Non-uniform filter banks	264
14.3.1 Subband coding	264
14.3.2 Two-channel quadrature mirror filter bank	266
14.3.3 Filter bank with perfect reconstruction	269
14.3.4 Perfect reconstruction two-channel FIR QMF bank	270
14.4 Transmultiplexers	271
15. Short-time spectral analysis	274
15.1 Uncertainty principle of resolution in time and in frequency	274
15.2 Gabor transform	275
15.3 Short-time Fourier transform of discrete signals	277
15.4 Short-time chirp spectrum	279
15.5 Using the EMD method for spectral analysis	282
15.5.1 Empirical mode decomposition	282
15.5.2 Example of applying the EMD to spectral analysis	284
16. Wavelet transform and filter banks	287
16.1 Mother wavelet, types of the wavelet transform	287
16.2 Dyadic discrete wavelet transform	290
16.3 Dyadic discrete wavelet transform with discrete time	291
16.4 Relation between continuous wavelet transform and analog filter bank	292
16.5 Relation between dyadic discrete wavelet transform and QMF bank of digital filters	294
16.6 Example of the QMF bank realized using the dyadic discrete wavelet transform with discrete time	298
16.7 Two-dimensional wavelet transform	300
16.7.1 Image data compression	300
16.7.2 Two-dimensional wavelet transform	302
17. Linear prediction analysis	303
17.1 Forward and backward linear prediction	303
17.1.1 Forward linear prediction	303
17.1.2 Backward linear prediction	306
17.1.3 Relation between linear prediction coefficients and reflection coefficients	307
17.1.4 Example of reflection coefficients calculation from linear prediction coefficients	308
17.2 Solution of the normal equations	309
17.3 Use of linear prediction to model the vocal tract	312

18. Homomorphic signal processing	316
18.1 Canonical representation of homomorphic systems	316
18.1.1 Generalized principle of superposition	316
18.1.2 Canonical representation of homomorphic systems	317
18.1.3 Multiplicative homomorphic systems	318
18.1.4 Homomorphic system for convolution	319
18.1.5 Note on terminology	320
18.2 Complex cepstrum	320
18.2.1 Definition of the complex cepstrum	320
18.2.2 Properties of the complex cepstrum	321
18.2.3 Example of a complex cepstrum	323
18.2.4 Complex cepstrum calculation using spectral analysis	323
18.3 Real cepstrum	325
18.3.1 Real cepstrum definition	325
18.3.2 Example of a real cepstrum	326
18.3.3 Approximation of exponential function using continued fraction expansion	327
18.3.4 Cepstral model of the vocal tract	328
19. Power spectral density	331
19.1 Definition of the power spectral density	331
19.2 Non-parametric methods for estimating the power spectral density	334
19.2.1 Bartlett method for averaging of periodograms	335
19.2.2 Welch method for averaging modified periodograms	336
19.2.3 Blackman and Tukey method for smoothing periodograms	337
19.2.4 Comparison of the properties of non-parametric methods for estimating the power spectral density	338
19.2.5 Example of calculating the power spectral density by the Welch method	339
19.3 Parametric methods for power spectrum density estimation	340
19.3.1 Parametric models – ARMA, AR and MA	340
19.3.2 Calculation of AR model parameters using autocorrelation sequence	342
19.3.3 Yule-Walker method for power spectrum estimation	343
19.3.4 Burg method for estimation of power spectral density	344
19.3.5 Selection of AR model order	345
19.4 Relation between power spectral density and cepstral coefficients	346
20. Higher-order spectra	349
20.1 Introduction	349
20.2 Cumulants and higher-order spectra	350
20.3 Properties of the bispectrum	351
20.4 Methods of bispectrum estimation	353
20.4.1 Indirect non-parametric estimation method of bispectrum	353
20.4.2 Choice of suitable window	354
20.4.3 Windows used in bispectrum estimation	355
20.5 Example of using bispectrum in practice	356
21. Examples of LTI discrete systems implementation	357
21.1 Design of an LTI discrete system based on analog prototype	357
21.2 Implementing first-order LTI discrete systems in microprocessors without hardware multiplier	361
21.3 Implementing first-order LTI discrete systems in microprocessors with hardware multiplier	362
21.4 Basic architecture and some properties of digital signal processors	363
21.5 Example of implementing a digital filter in a digital signal processor with VLIW architecture	367

22. Random variables and stochastic processes	378
22.1 Random variables, continuous-time and discrete-time stochastic processes	378
22.2 Moments and other characteristics	381
22.3 Example of moments of a discrete random variable	384
22.4 Covariance and random vectors	386
22.5 Stationary and ergodic stochastic processes	386
22.6 Markov chains	389
22.6.1 Discrete time Markov chains	389
22.6.2 Continuous time Markov chains	396
23. Mathematical statistics	401
23.1 Historical introduction	401
23.2 Statistical model and random sample	401
23.3 Statistic and its properties	402
23.4 Point estimations of parameters and their functions	406
23.5 Interval estimates of parameters	409
23.6 Statistical hypothesis testing	412
23.7 Parametric tests	414
23.8 Nonparametric tests	427
23.9 Goodness of fit tests	431
23.10 Statistical binary classification	438
23.10.1 Evaluation of binary classifiers	438
References	444
Index	447