

Synthesis Lectures on Mathematics & Statistics

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Arturo Portnoy

The Mathematics of Music and Art

This book explores the relationships between music, the sciences, and mathematics, both ancient and modern, with a focus on the big picture for a general audience as opposed to delving into very technical details. The language of music is deciphered through the language of mathematics. Readers are shown how apparently unrelated areas of knowledge complement each other and in fact propel each other's advancement. The presentation as well as the collection of topics covered throughout is unique and serves to encourage exploration and also, very concretely, illustrates the cross- and multidisciplinary nature of knowledge. Inspired by an introductory, multidisciplinary course, the author explores the relationships between the arts, sciences, and mathematics in the realm of music. The book has no prerequisites; rather it aims to give a broad overview and achieve the integration of the three presented themes. Mathematical tools are introduced and used to explain various aspects of music theory, and the author illustrates how, without mathematics, music could not have been developed.

This book:

- Discusses the relationships between the arts, sciences, and mathematics in the realm of music
- Explores the language of music as deciphered through the language of mathematics
- Shows readers how apparently unrelated areas of knowledge complement each other and propel each other's advancement

About the Author

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1	Preliminaries	1
1.1	Rational and Irrational Numbers	1
1.2	Logarithms and Exponentials	4
1.2.1	Exponential Function	7
1.2.2	Exponential Equations	8
1.2.3	Logarithmic Function	9
1.3	Pitch, the Musical Scale, and the Piano	11
1.4	Musical Notation, Rhythm	12
1.5	Consonance and Dissonance	14
1.6	Key, Melody, Chords, and Harmony	16
2	The Pigeonhole Principle	19
2.1	Pigeons and Pigeonholes	19
2.2	Rational Approximations	22
3	Continued Fractions	27
3.1	Finite Continued Fractions	27
3.2	Infinite Continued Fractions	28
3.3	The Golden Ratio	29
3.4	Convergents	33
4	Pythagoras and Music	35
4.1	The Monochord	35
4.2	The Circle of Fifths and the Musical Scale	41
4.3	Why Does the Occidental Chromatic Scale Have 12 Notes?	44
5	Temperaments of the Scale	49
5.1	The Pythagorean Scale	51
5.2	Just Intonation	51
5.3	The Equally Tempered Scale	56
5.4	Comparison	56
5.5	Project	60

6	Patterns in Music	61
6.1	Rhythmic Patterns	62
6.2	Patterns in Melody and Harmony	65
6.2.1	The Diatonic Major Scale	65
6.2.2	The Relative Minor Scale	66
6.2.3	Triads and Chords	67
6.2.4	Chord Inversions	68
6.2.5	Modular Arithmetic in the Scale	69
6.3	Patterns in Musical Composition	69
6.3.1	Transposition	77
6.3.2	Composition	78
6.4	Symmetry in Harmony	80
6.5	Symmetry in Composition	81
6.6	Project	82
7	Analysis and Synthesis of Music	83
7.1	Harmonic Oscillations	83
7.2	Fourier Series	85
7.3	Analysis of Sound	87
7.4	Sound Synthesis	92
7.5	The Acoustic Envelope	93
8	Beats, Resonance, and Tuning	97
8.1	Beats	97
8.2	Resonance	99
8.3	Tuning	99
9	Digital Music and Information	101
9.1	Binary Representation of Numbers	101
9.2	Analog Versus Digital	102
9.3	Digitalization of Sound and Music	103
10	Last Thoughts	111
	Appendix A: Mathematical Induction	115
	Appendix B: ¿Do Convergents Actually Converge?	117
	Appendix C: Fibonacci and the Golden Ratio	121
	Further Reading	125
	Index	127