Contents

Introduction xiii

Notation xvi

1 Propositional Logic 1

1.1 Boolean Functions and Formulas 2
1.2 Semantic Equivalence and Normal Forms 9
1.3 Tautologies and Logical Consequence 14
1.4 A Complete Calculus for $\models$ 18
1.5 Applications of the Compactness Theorem 25
1.6 Hilbert Calculi 29

2 Predicate Logic 33

2.1 Mathematical Structures 34
2.2 Syntax of Elementary Languages 43
2.3 Semantics of Elementary Languages 49
2.4 General Validity and Logical Equivalence 58
2.5 Logical Consequence and Theories 62
2.6 Explicit Definitions—Expanding Languages 67

3 Gödel's Completeness Theorem 71

3.1 A Calculus of Natural Deduction 72
3.2 The Completeness Proof 76
3.3 First Applications—Nonstandard Models 81
3.4 ZFC and Skolem's Paradox 87
3.5 Enumerability and Decidability 92
3.6 Complete Hilbert Calculi 95
3.7 First-Order Fragments and Extensions 99

4 The Foundations of Logic Programming **105**
4.1 Term Models and Horn Formulas 106
4.2 Propositional Resolution 112
4.3 Unification 119
4.4 Logic Programming 122
4.5 Proof of the Main Theorem 129

5 Elements of Model Theory **131**
5.1 Elementary Extensions 132
5.2 Complete and κ-Categorical Theories 137
5.3 Ehrenfeucht's game 142
5.4 Embedding and Characterization Theorems 145
5.5 Model Completeness 151
5.6 Quantifier Elimination 157
5.7 Reduced Products and Ultraproducts 163

6 Incompleteness and Undecidability **167**
6.1 Recursive and Primitive Recursive Functions 169
6.2 Arithmetization 176
6.3 Representability of Arithmetical Predicates 182
6.4 The Representability Theorem 189
6.5 The Theorems of Gödel, Tarski, Church 194
6.6 Transfer by Interpretation 200
6.7 The Arithmetical Hierarchy 205

7 On the Theory of Self-Reference **209**
7.1 The Derivability Conditions 210
7.2 The Theorems of Gödel and Löb 217
7.3 The Provability Logic G 221
7.4 The Modal Treatment of Self-Reference 223
7.5 A Bimodal Provability Logic for PA 226
7.6 Modal Operators in ZFC 228

Hints to the Exercises **231**

Literature **241**

Index of Terms and Names **247**

Index of Symbols **255**