

## TABLE OF CONTENTS

|  |           |
|--|-----------|
| <b>1. INTRODUCTION.....</b>  | <b>1</b>  |
| <b>    1.1 DIGITAL HOLOGRAPHY AND EVOLUTION OF IMAGING<br/>    TECHNIQUES.....</b>                                       | <b>1</b>  |
| <b>    1.2 CONTENTS OF THIS BOOK .....</b>   | <b>5</b>  |
| <b>    REFERENCES .....</b>  | <b>10</b> |
| <b>2. OPTICAL SIGNALS AND TRANSFORMS.....</b>  | <b>11</b> |
| <b>    2.1 MATHEMATICAL MODELS OF OPTICAL SIGNALS.....</b>   | <b>11</b> |
| 2.1.1 Primary definitions and classification.....  | 11        |
| 2.1.2 Signal space .....   | 13        |
| 2.1.3 Linear signal space, basis functions and signal representation as<br>expansion over a set of basis functions ..... | 17        |
| 2.1.4 Integral representation of signals.....  | 22        |
| <b>    2.2 SIGNAL TRANSFORMATIONS .....</b>  | <b>24</b> |
| <b>    2.3 IMAGING SYSTEMS AND INTEGRAL TRANSFORMS.....</b>  | <b>27</b> |
| 2.3.1 Direct imaging: Convolution integral .....   | 27        |
| 2.3.2 Imaging in Fourier domain: Holography and diffraction integrals....  | 30        |
| <b>    2.4 FOURIER TRANSFORM AND ITS DERIVATIVES .....</b>   | <b>39</b> |
| 2.4.1 Properties of integral Fourier Transform .....   | 39        |
| 2.4.2 Special cases of the integral Fourier transform: integral Cosine,<br>Hartley, Hankel and Mellin transforms.....    | 44        |
| 2.4.3 Integral Fresnel and related transform .....   | 49        |
| 2.4.4 Hilbert transform .....  | 53        |
| <b>    2.5 IMAGING FROM PROJECTIONS: RADON AND ABEL<br/>    TRANSFORMS .....</b>   | <b>57</b> |
| <b>    2.6 MULTI RESOLUTION IMAGING: WAVELET TRANSFORMS ...</b>  | <b>61</b> |

|   |            |
|---|------------|
| <b>2.7 SLIDING WINDOW TRANSFORMS AND "TIME-FREQUENCY"<br/>(SPACE-TRANSFORM) SIGNAL REPRESENTATION .....</b>                           | <b>63</b>  |
| 2.7.1 Transforms in sliding window (windowed transforms)  | 63         |
| 2.7.2 Sliding window Fourier Transform .....  | 53         |
| 2.7.3 Sliding window and wavelet transforms as signal sub-band<br>decomposition .....   | 64         |
| <b>2.8 STOCHASTIC TRANSFORMATIONS AND STATISTICAL<br/>MODELS .....</b>  | <b>67</b>  |
| 2.8.1 Additive, multiplicative, Poisson noise and impulse noise models ....   | 67         |
| 2.8.2 Speckle noise model .....   | 72         |
| <b>References .....</b>   | <b>77</b>  |
| <b>3. DIGITAL REPRESENTATION OF SIGNALS .....</b>   | <b>79</b>  |
| <b>3.1 PRINCIPLES OF SIGNAL DIGITIZATION .....</b>  | <b>79</b>  |
| <b>3.2 SIGNAL DISCRETIZATION AS EXPANSION OVER A SET OF<br/>BASIS FUNCTIONS. TYPICAL BASIS FUNCTIONS AND<br/>CLASSIFICATION .....</b> | <b>80</b>  |
| 3.2.1 Shift (convolution) basis functions .....   | 81         |
| 3.2.2 Scale (multiplicative) basis functions .....  | 85         |
| 3.2.3 Wavelets .....  | 92         |
| 3.2.4 Discrete bases .....  | 94         |
| 3.2.5 Optimality of bases. Karhunen-Loeve, Hotelling and Singular Values<br>Decomposition Transforms .....                            | 97         |
| 3.2.6 Two dimensional and multi dimensional bases .....   | 103        |
| <b>3.3 SHIFT (CONVOLUTION) BASES FUNCTIONS AND SAMPLING<br/>THEOREM .....</b>   | <b>106</b> |
| 3.3.1 1-D sampling theorem .....  | 106        |
| 3.3.2 Sampling 2-D and multidimensional signals .....   | 113        |
| 3.3.3 Sampling artifacts: qualitative analysis .....  | 120        |
| 3.3.4 Sampling artifacts: quantitative analysis .....   | 124        |
| <b>3.4 MULTI-RESOLUTION SAMPLING .....</b>  | <b>129</b> |
| <b>3.5 UNCONVENTIONAL DIGITAL IMAGING METHODS .....</b>   | <b>131</b> |
| <b>3.6 PRINCIPLES OF SIGNAL SCALAR QUANTIZATION .....</b>   | <b>133</b> |
| 3.6.1 Optimal homogeneous non-uniform quantization .....  | 133        |

|   |            |
|---|------------|
| 3.6.2 Quantization in digital holography .....                                    | 143        |
| <b>3.7 BASICS OF SIGNAL CODING AND DATA COMPRESSION ....</b>                      | <b>147</b> |
| 3.1.1 Signal rate distortion function, entropy and statistical coding.....        | 147        |
| 3.7.2 Image compression methods: a review .....                                   | 152        |
| <b>4. DIGITAL REPRESENTATION OF SIGNAL TRANSFORMATIONS.....</b>                   | <b>161</b> |
| <b>4.1 THE PRINCIPLES .....</b>   | <b>161</b> |
| <b>4.2 DISCRETE REPRESENTATION OF CONVOLUTION INTEGRAL. DIGITAL FILTERS .....</b> | <b>170</b> |
| <b>4.3 DISCRETE REPRESENTATION OF FOURIER INTEGRAL TRANSFORM. .....</b>           | <b>170</b> |
| 4.3.1 Discrete Fourier Transforms .....   | 170        |
| 4.3.2 Properties of Discrete Fourier Transforms .....                             | 175        |
| 4.3.3 Discrete cosine and sine transforms .....                                   | 189        |
| 4.3.4 Lapped Transforms .....   | 195        |
| <b>4.4 DISCRETE REPRESENTATION OF FRESNEL INTEGRAL TRANSFORM .....</b>            | <b>199</b> |
| 4.4.1 Discrete Fresnel Transform.....   | 199        |
| 4.4.2 Properties of Discrete Fresnel Transforms .....                             | 205        |
| <b>References .....</b>   | <b>209</b> |
| <b>5. METHODS AND ALGORITHMS OF DIGITAL FILTERING.....</b>                        | <b>211</b> |
| <b>5.1 FILTERING IN SIGNAL DOMAIN .....</b>                                       | <b>211</b> |
| 5.1.1 Transversal and recursive filtering.....                                    | 211        |
| 5.1.2 Separable, cascade, parallel and parallel recursive filtering.....          | 213        |
| 5.1.3 Recursive algorithms of sliding window DFT and DCT/DcST .....               | 220        |
| 5.1.4 Border processing in digital filtering.....                                 | 223        |
| <b>5.2 FILTERING IN TRANSFORM DOMAIN .....</b>                                    | <b>226</b> |
| 5.2.1 DFT and cyclic convolution. Border processing.....                          | 226        |
| 5.2.2 Signal convolution in the DCT domain.....                                   | 227        |

|  |            |
|--|------------|
| <b>5.3 COMBINED ALGORITHMS FOR COMPUTING DFT AND DCT OF REAL VALUED SIGNALS .....</b>  | <b>231</b> |
| 5.3.1 Combined algorithms for computing DFT of signals with real samples .....         | 231        |
| 5.3.2 Combined algorithm for computing the DCT via FFT .....                           | 234        |
| <b>References .....</b>  | <b>237</b> |
| <b>6. FAST ALGORITHMS .....</b>  | <b>239</b> |
| <b>    6.1 THE PRINCIPLE OF FAST FOURIER TRANSFORMS.....</b>                           | <b>239</b> |
| <b>    6.2 MATRIX TECHNIQUES IN FAST TRANSFORMS.....</b>                               | <b>243</b> |
| <b>    6.3 TRANSFORMS AND THEIR FAST ALGORITHMS IN MATRIX REPRESENTATION .....</b>     | <b>249</b> |
| 6.3.1 Walsh-Hadamard transform .....   | 249        |
| 6.3.2 Walsh-Paley Transform and Bit Reversal Permutation.....                          | 250        |
| 6.3.3 Walsh Transform and Binary-to-Gray Code and Hadamard Permutations .....          | 252        |
| 6.3.4 Haar Transform .....   | 254        |
| 6.3.5 Discrete Fourier Transform.....  | 255        |
| 6.3.6 DCT and DcST .....   | 260        |
| <b>    6.4 PRUNED ALGORITHMS .....</b>   | <b>264</b> |
| <b>    6.5 QUANTIZED DFT .....</b>   | <b>269</b> |
| <b>REFERENCES .....</b>  | <b>274</b> |
| <b>7. STATISTICAL METHODS AND ALGORITHMS.....</b>                                      | <b>275</b> |
| <b>    7.1 MEASURING SIGNAL STATISTICAL CHARACTERISTICS....</b>                        | <b>275</b> |
| 7.1.1 Measuring probability distribution and its moments and order statistics .....    | 275        |
| 7.1.2 Measuring signal correlation functions and spectra.....                          | 280        |
| 7.1.3 Measuring parameters of random interferences in sensor and imaging systems ..... | 284        |
| <b>    7.2 DIGITAL STATISTICAL MODELS AND MONTE CARLO METHODS .....</b>                | <b>291</b> |

|   |            |
|---|------------|
| 7.2.1 Principles of generating pseudo-random numbers. Generating independent uniformly distributed pseudo-random numbers.....         | 291        |
| 7.2.2 Generating pseudo-random numbers with special statistical properties .....  | 292        |
| 7.2.3 Generating pseudo-random images .....   | 297        |
| 7.2.4 Generating correlated phase masks. Kinoform and programmed diffuser holograms .....   | 300        |
| <b>7.3 STATISTICAL (MONTE CARLO) SIMULATION. CASE STUDY: SPECKLE NOISE PHENOMENA IN COHERENT IMAGING AND DIGITAL HOLOGRAPHY .....</b> | <b>305</b> |
| 7.3.1 Computer model.....   | 306        |
| 7.3.2 Simulation results.....   | 307        |
| <b>References .....</b>   | <b>312</b> |
| <b>8. SENSOR SIGNAL PERFECTING, IMAGE RESTORATION, RECONSTRUCTION AND ENHANCEMENT .....</b>   | <b>313</b> |
| <b>8.1 MATHEMATICAL MODELS OF IMAGING SYSTEMS .....</b>   | <b>313</b> |
| <b>8.2 LINEAR FILTERS FOR IMAGE RESTORATION.....</b>  | <b>315</b> |
| 8.2.1 Transform domain MSE optimal scalar Wiener Filters.....   | 315        |
| 8.2.2 Empirical Wiener filters for image denoising .....  | 317        |
| 8.2.3 Image deblurring, inverse filters and aperture correction.....  | 323        |
| <b>8.3 SLIDING WINDOW TRANSFORM DOMAIN ADAPTIVE SIGNAL RESTORATION .....</b>  | <b>328</b> |
| 8.3.1 Local adaptive filtering .....  | 328        |
| 8.3.2 Sliding window transform domain DCT filtering .....   | 330        |
| 8.3.3 Hybrid DCT/wavelet filtering .....  | 334        |
| <b>8.4 MULTI-COMPONENT IMAGE RESTORATION.....</b>   | <b>340</b> |
| <b>8.5 FILTERING IMPULSE NOISE .....</b>  | <b>343</b> |
| <b>8.6 METHODS FOR CORRECTING GRAY SCALE NONLINEAR DISTORTIONS .....</b>  | <b>348</b> |
| <b>8.7 IMAGE RECONSTRUCTION .....</b>   | <b>353</b> |

|   |            |
|---|------------|
| <b>8.8 IMAGE ENHANCEMENT.....</b>   | <b>361</b> |
| 8.8.1 Image enhancement as an image processing task. Classification of image enhancement methods .....  | 361        |
| 8.8.2 Gray level histogram modification methods .....   | 362        |
| 8.8.3 Image spectra modification methods.....   | 367        |
| 8.8.4 Using color, stereo and dynamical vision for image enhancement...   | 369        |
| <b>References .....</b>   | <b>371</b> |
| <b>9. IMAGE RESAMPLING AND GEOMETRICAL TRANSFORMATIONS .....</b>  | <b>373</b> |
| <b>9.1 PRINCIPLES OF IMAGE RESAMPLING .....</b>   | <b>373</b> |
| <b>9.2 NEAREST NEIGHBOR, LINEAR AND SPLINE INTERPOLATION METHODS .....</b>  | <b>376</b> |
| <b>9.3 ALGORITHMS OF DISCRETE SINC-INTERPOLATION .....</b>  | <b>380</b> |
| 9.3.1 Discrete sinc-interpolation by zero padding signal DFT spectrum..   | 380        |
| 9.3.2 DFT based discrete sinc-interpolation algorithm for signal arbitrary translation.....   | 382        |
| 9.3.3 DCT based discrete sinc-interpolation algorithm for signal arbitrary translation .....  | 386        |
| 9.3.4 Sliding window adaptive discrete sinc-interpolation algorithms .....  | 388        |
| <b>9.4 Application examples.....</b>  | <b>395</b> |
| 9.4.1 Signal and image resizing and localization with sub-pixel accuracy  | 395        |
| 9.4.2 Fast algorithm for image rotation with discrete-sinc-interpolation..  | 398        |
| 9.4.3 Signal differentiating and filtered back projection method for tomographic reconstruction.....  | 400        |
| 9.4.4 Signal integrating and reconstructing surfaces from their slope .....   | 403        |
| 9.4.5 Polar-to-Cartesian coordinate conversion, interpolation of DFT spectra of projections and the direct Fourier method of image reconstruction from projections..... | 407        |
| <b>References .....</b>   | <b>410</b> |

|   |            |
|---|------------|
| <b>10. SIGNAL PARAMETER ESTIMATION AND MEASUREMENT.</b>   | 51         |
| <b>OBJECT LOCALIZATION .....</b>  | <b>411</b> |
| <b>    10.1 PROBLEM FORMULATION. OPTIMAL STATISTICAL<br/>ESTIMATES .....</b>                                      | <b>411</b> |
| <b>    10.2 LOCALIZATION OF AN OBJECT IN THE PRESENCE OF<br/>ADDITIVE WHITE GAUSSIAN NOISE .....</b>              | <b>414</b> |
| 10.2.1 Optimal localization device. Correlator and matched filter .....   | 414        |
| 10.2.2 Computer and optical implementations of the matched filter<br>correlator .....                             | 417        |
| 10.2.3 Performance of optimal estimators: normal and anomalous<br>localization errors.....                        | 419        |
| <b>    10.3 PERFORMANCE OF THE OPTIMAL LOCALIZATION DEVICE</b><br>.....   | <b>421</b> |
| 10.3.1 Distribution density and variance of normal errors.....  | 421        |
| 10.3.2 Illustrative examples .....  | 431        |
| 10.3.3 Localization accuracy for non-optimal localization devices. ....   | 436        |
| 10.3.4 Localization reliability. Probability of anomalous localization errors<br>.....                            | 439        |
| <b>    10.4 LOCALIZATION OF AN OBJECT IN THE PRESENCE OF<br/>ADDITIVE CORRELATED GAUSSIAN NOISE .....</b>         | <b>443</b> |
| 10.4.1 Localization of a target object in the presence of non-white<br>(correlated) additive Gaussian noise ..... | 443        |
| 10.4.2 Localization accuracy of the optimal filter.....   | 445        |
| <b>    10.5 OPTIMAL LOCALIZATION IN COLOR AND MULTI<br/>COMPONENT IMAGES .....</b>                                | <b>449</b> |
| 10.5.1 Optimal localization device.....   | 449        |
| 10.5.2 Localization accuracy and reliability.....   | 451        |
| 10.5.3 Optimal localization in multi component images with correlated<br>noise. ....                              | 452        |
| <b>    10.6 OBJECT LOCALIZATION IN THE PRESENCE OF MULTIPLE<br/>NONOVERLAPPING NON-TARGET OBJECTS .....</b>       | <b>454</b> |
| <b>References .....</b>   | <b>459</b> |

|  |            |
|--|------------|
| <b>11. TARGET LOCATION IN CLUTTER .....</b>  | <b>461</b> |
| <b>    11.1 PROBLEM FORMULATION .....</b>  | <b>461</b> |
| <b>    11.2 LOCALIZATION OF PRECISELY KNOWN OBJECTS:<br/>SPATIALLY HOMOGENEOUS OPTIMALITY CRITERION .....</b>            | <b>465</b> |
| 11.2.1 Optimal adaptive correlator.....  | 465        |
| 11.2.2 Implementation issues, practical recommendations and illustrative<br>examples .....                               | 469        |
| <b>    11.3 LOCALIZATION OF INEXACTLY KNOWN OBJECT:<br/>SPATIALLY HOMOGENEOUS CRITERION.....</b>                         | <b>474</b> |
| <b>    11.4 LOCALIZATION METHODS FOR SPATIALLY<br/>INHOMOGENEOUS CRITERIA .....</b>                                      | <b>478</b> |
| <b>    11.5 OBJECT LOCALIZATION AND IMAGE BLUR .....</b>   | <b>483</b> |
| <b>    11.6 OBJECT LOCALIZATION AND EDGE DETECTION.<br/>SELECTION OF REFERENCE OBJECTS FOR TARGET TRACKING<br/>.....</b> | <b>485</b> |
| <b>    11.7 OPTIMAL ADAPTIVE CORRELATOR AND OPTICAL<br/>CORRELATORS .....</b>  | <b>490</b> |
| 11.7.1 Optical correlators with improved discrimination capability<br>.....  | 490        |
| 11.7.2 Computer and optical implementations of the optimal adaptive<br>correlator .....                                  | 493        |
| <b>    11.8 TARGET LOCATING IN COLOR AND MULTI COMPONENT<br/>IMAGES .....</b>  | <b>499</b> |
| 11.8.1 Theoretical framework .....   | 499        |
| 11.8.2 Separable component-wise implementations of the optimal adaptive<br>multi-component correlator.....               | 501        |
| <b>References .....</b>  | <b>507</b> |
| <b>12. NONLINEAR FILTERS IN SIGNAL/IMAGE PROCESSING .....</b>  | <b>509</b> |
| <b>    12.1 CLASIFICATION PRINCIPLES .....</b>   | <b>510</b> |
| 12.1.1 Main assumptions and definitions .....  | 510        |

|                         |   |            |
|-------------------------|---|------------|
| 12.1.2                  | Typical signal attributes .....   | 511        |
| 12.1.3                  | Estimation operations.....  | 512        |
| 12.1.4                  | Neighborhood building operations.....   | 515        |
| <b>12.2</b>             | <b>FILTER CLASSIFICATION TABLES .....</b>                                       | <b>519</b> |
| 12.2.1                  | Transferential filters.....   | 519        |
| 12.2.2                  | Iterative filtering.....  | 522        |
| 12.2.3                  | Multiple branch parallel, cascade and recursive filters .....                   | 523        |
| 12.2.4                  | Miscellaneous new filters.....  | 524        |
| <b>12.3</b>             | <b>PRACTICAL EXAMPLES .....</b>   | <b>527</b> |
| 12.3.1                  | Image denoising .....   | 527        |
| 12.3.2                  | Image enhancement.....  | 531        |
| 12.3.3                  | Image segmentation and edge detection.....                                      | 535        |
| 12.3.4                  | Implementation issues .....   | 538        |
| <b>References .....</b> |   | <b>540</b> |
| <b>13.</b>              | <b>COMPUTER GENERATED HOLOGRAMS .....</b>                                       | <b>541</b> |
| <b>13.1</b>             | <b>MATHEMATICAL MODELS.....</b>   | <b>541</b> |
| <b>13.2</b>             | <b>METHODS FOR ENCODING AND RECORDING COMPUTER<br/>GENERATED HOLOGRAMS.....</b> | <b>548</b> |
| <b>13.3</b>             | <b>RECONSTRUCTION OF COMPUTER GENERATED<br/>HOLOGRAMS .....</b>                 | <b>565</b> |
| <b>References.....</b>  |   | <b>575</b> |