

Contents

		<i>page</i>
Preface	Additional reading	xv
1 Vector algebra		1
1.1 Preliminaries		1
1.2 Coordinate system invariance		4
1.3 Vector multiplication		9
1.4 Useful products of vectors		12
1.5 Linear vector spaces		13
1.6 Focus: periodic media and reciprocal lattice vectors		17
1.7 Additional reading		24
1.8 Exercises		24
2 Vector calculus		28
2.1 Introduction		28
2.2 Vector integration		29
2.3 The gradient, ∇		35
2.4 Divergence, $\nabla \cdot$		37
2.5 The curl, $\nabla \times$		41
2.6 Further applications of ∇		43
2.7 Gauss' theorem (divergence theorem)		45
2.8 Stokes' theorem		47
2.9 Potential theory		48
2.10 Focus: Maxwell's equations in integral and differential form		51
2.11 Focus: gauge freedom in Maxwell's equations		57
2.12 Additional reading		60
2.13 Exercises		60
3 Vector calculus in curvilinear coordinate systems		64
3.1 Introduction: systems with different symmetries		64
3.2 General orthogonal coordinate systems		65
3.3 Vector operators in curvilinear coordinates		69
3.4 Cylindrical coordinates		73
Focus: X-ray diffraction from crystals and thin films		73
Focus: Fraunhofer diffraction from periodic structures		85

3.5	Spherical coordinates	76
3.6	Exercises	79
4	Matrices and linear algebra	83
4.1	Introduction: Polarization and Jones vectors	83
4.2	Matrix algebra	88
4.3	Systems of equations, determinants, and inverses	93
4.4	Orthogonal matrices	102
4.5	Hermitian matrices and unitary matrices	105
4.6	Diagonalization of matrices, eigenvectors, and eigenvalues	107
4.7	Gram–Schmidt orthonormalization	115
4.8	Orthonormal vectors and basis vectors	118
4.9	Functions of matrices	120
4.10	Focus: matrix methods for geometrical optics	120
4.11	Additional reading	133
4.12	Exercises	133
5	Advanced matrix techniques and tensors	139
5.1	Introduction: Foldy–Lax scattering theory	139
5.2	Advanced matrix terminology	142
5.3	Left–right eigenvalues and biorthogonality	143
5.4	Singular value decomposition	146
5.5	Other matrix manipulations	153
5.6	Tensors	159
5.7	Additional reading	174
5.8	Exercises	174
6	Distributions	177
6.1	Introduction: Gauss' law and the Poisson equation	177
6.2	Introduction to delta functions	181
6.3	Calculus of delta functions	184
6.4	Other representations of the delta function	185
6.5	Heaviside step function	187
6.6	Delta functions of more than one variable	188
6.7	Additional reading	192
6.8	Exercises	192
7	Infinite series	195
7.1	Introduction: the Fabry–Perot interferometer	195
7.2	Sequences and series	198
7.3	Series convergence	201
7.4	Series of functions	210
7.5	Taylor series	213
7.6	Taylor series in more than one variable	218
7.7	Power series	220
7.8	Focus: convergence of the Born series	221

7.9	Additional reading	226
7.10	Exercises	226
8	Fourier series	230
8.1	Introduction: diffraction gratings	230
8.2	Real-valued Fourier series	233
8.3	Examples	236
8.4	Integration range of the Fourier series	239
8.5	Complex-valued Fourier series	239
8.6	Properties of Fourier series	240
8.7	Gibbs phenomenon and convergence in the mean	243
8.8	Focus: X-ray diffraction from crystals	246
8.9	Additional reading	249
8.10	Exercises	249
9	Complex analysis	252
9.1	Introduction: electric potential in an infinite cylinder	252
9.2	Complex algebra	254
9.3	Functions of a complex variable	258
9.4	Complex derivatives and analyticity	261
9.5	Complex integration and Cauchy's integral theorem	265
9.6	Cauchy's integral formula	269
9.7	Taylor series	271
9.8	Laurent series	273
9.9	Classification of isolated singularities	276
9.10	Branch points and Riemann surfaces	278
9.11	Residue theorem	285
9.12	Evaluation of definite integrals	288
9.13	Cauchy principal value	297
9.14	Focus: Kramers–Kronig relations	299
9.15	Focus: optical vortices	302
9.16	Additional reading	308
9.17	Exercises	308
10	Advanced complex analysis	312
10.1	Introduction	312
10.2	Analytic continuation	312
10.3	Stereographic projection	316
10.4	Conformal mapping	325
10.5	Significant theorems in complex analysis	332
10.6	Focus: analytic properties of wavefields	340
10.7	Focus: optical cloaking and transformation optics	345
10.8	Exercises	348
11	Fourier transforms	350
11.1	Introduction: Fraunhofer diffraction	350

11.2	The Fourier transform and its inverse	352
11.3	Examples of Fourier transforms	354
11.4	Mathematical properties of the Fourier transform	358
11.5	Physical properties of the Fourier transform	365
11.6	Eigenfunctions of the Fourier operator	372
11.7	Higher-dimensional transforms	373
11.8	Focus: spatial filtering	375
11.9	Focus: angular spectrum representation	377
11.10	Additional reading	382
11.11	Exercises	383
12	Other integral transforms	386
12.1	Introduction: the Fresnel transform	386
12.2	Linear canonical transforms	391
12.3	The Laplace transform	395
12.4	Fractional Fourier transform	400
12.5	Mixed domain transforms	402
12.6	The wavelet transform	406
12.7	The Wigner transform	409
12.8	Focus: the Radon transform and computed axial tomography (CAT)	410
12.9	Additional reading	416
12.10	Exercises	416
13	Discrete transforms	419
13.1	Introduction: the sampling theorem	419
13.2	Sampling and the Poisson sum formula	423
13.3	The discrete Fourier transform	427
13.4	Properties of the DFT	430
13.5	Convolution	432
13.6	Fast Fourier transform	433
13.7	The z -transform	437
13.8	Focus: z -transforms in the numerical solution of Maxwell's equations	445
13.9	Focus: the Talbot effect	449
13.10	Exercises	456
14	Ordinary differential equations	458
14.1	Introduction: the classic ODEs	458
14.2	Classification of ODEs	459
14.3	Ordinary differential equations and phase space	460
14.4	First-order ODEs	469
14.5	Second-order ODEs with constant coefficients	474
14.6	The Wronskian and associated strategies	476
14.7	Variation of parameters	478
14.8	Series solutions	480
14.9	Singularities, complex analysis, and general Frobenius solutions	481

14.10	Integral transform solutions	485
14.11	Systems of differential equations	486
14.12	Numerical analysis of differential equations	488
14.13	Additional reading	501
14.14	Exercises	501
15	Partial differential equations	505
15.1	Introduction: propagation in a rectangular waveguide	505
15.2	Classification of second-order linear PDEs	508
15.3	Separation of variables	517
15.4	Hyperbolic equations	519
15.5	Elliptic equations	525
15.6	Parabolic equations	530
15.7	Solutions by integral transforms	534
15.8	Inhomogeneous problems and eigenfunction solutions	538
15.9	Infinite domains; the d'Alembert solution	539
15.10	Method of images	544
15.11	Additional reading	545
15.12	Exercises	545
16	Bessel functions	550
16.1	Introduction: propagation in a circular waveguide	550
16.2	Bessel's equation and series solutions	552
16.3	The generating function	555
16.4	Recurrence relations	557
16.5	Integral representations	560
16.6	Hankel functions	564
16.7	Modified Bessel functions	565
16.8	Asymptotic behavior of Bessel functions	566
16.9	Zeros of Bessel functions	567
16.10	Orthogonality relations	569
16.11	Bessel functions of fractional order	572
16.12	Addition theorems, sum theorems, and product relations	576
16.13	Focus: nondiffracting beams	579
16.14	Additional reading	582
16.15	Exercises	582
17	Legendre functions and spherical harmonics	585
17.1	Introduction: Laplace's equation in spherical coordinates	585
17.2	Series solution of the Legendre equation	587
17.3	Generating function	589
17.4	Recurrence relations	590
17.5	Integral formulas	592
17.6	Orthogonality	594
17.7	Associated Legendre functions	597

17.8	Spherical harmonics	602
17.9	Spherical harmonic addition theorem	605
17.10	Solution of PDEs in spherical coordinates	608
17.11	Gegenbauer polynomials	610
17.12	Focus: multipole expansion for static electric fields	611
17.13	Focus: vector spherical harmonics and radiation fields	614
17.14	Exercises	618
18	Orthogonal functions	622
18.1	Introduction: Sturm–Liouville equations	622
18.2	Hermite polynomials	627
18.3	Laguerre functions	641
18.4	Chebyshev polynomials	650
18.5	Jacobi polynomials	654
18.6	Focus: Zernike polynomials	655
18.7	Additional reading	662
18.8	Exercises	662
19	Green’s functions	665
19.1	Introduction: the Huygens–Fresnel integral	665
19.2	Inhomogeneous Sturm–Liouville equations	669
19.3	Properties of Green’s functions	674
19.4	Green’s functions of second-order PDEs	676
19.5	Method of images	685
19.6	Modal expansion of Green’s functions	689
19.7	Integral equations	693
19.8	Focus: Rayleigh–Sommerfeld diffraction	701
19.9	Focus: dyadic Green’s function for Maxwell’s equations	704
19.10	Focus: scattering theory and the Born series	709
19.11	Exercises	712
20	The calculus of variations	715
20.1	Introduction: principle of Fermat	715
20.2	Extrema of functions and functionals	718
20.3	Euler’s equation	721
20.4	Second form of Euler’s equation	727
20.5	Calculus of variations with several dependent variables	730
20.6	Calculus of variations with several independent variables	732
20.7	Euler’s equation with auxiliary conditions: Lagrange multipliers	734
20.8	Hamiltonian dynamics	739
20.9	Focus: aperture apodization	742
20.10	Additional reading	745
20.11	Exercises	745
21	Asymptotic techniques	748
21.1	Introduction: foundations of geometrical optics	748

21.2	Definition of an asymptotic series	753
21.3	Asymptotic behavior of integrals	756
21.4	Method of stationary phase	763
21.5	Method of steepest descents	766
21.6	Method of stationary phase for double integrals	771
21.7	Additional reading	772
21.8	Exercises	773
<i>Appendix A</i> The gamma function		775
A.1	Definition	775
A.2	Basic properties	776
A.3	Stirling's formula	778
A.4	Beta function	779
A.5	Useful integrals	780
<i>Appendix B</i> Hypergeometric functions		783
B.1	Hypergeometric function	784
B.2	Confluent hypergeometric function	785
B.3	Integral representations	785
<i>References</i>		787
<i>Index</i>		793

The Fourier transform learned in math class is internally created differently than the Fourier transform used in, say, Fraunhofer diffraction. The end result is that one student effectively learns the same topic twice, and is unable to use the intuition learned in a physics class to help aid in mathematical understanding, or to use the techniques learned in math class to formulate and solve physical problems.

To try and correct for this, I began to devote special lectures to the consequences of the mathematics students were studying. Lectures on complex analysis would be followed by discussions of the analytic properties of wavefields and the Kramers-Kronig relations. Lectures on infinite series could be taught by the discussion of the Furry-Perci interference.

Students in my classes were uniformly dissatisfied with the standard textbooks. Part of this dissatisfaction arises from the broad topics from which examples are drawn: quantum physics, field theory, general relativity, optics, mechanics, and thermodynamics, to name a few. Even the most dedicated theoretical physics students do not have a great physical intuition about all these subfields, and consequently many of the examples are not immediately in their mind when problems in abstract mathematics.

Given that graduate students are studying optics, this forced my attention on methods directly related to optical science. Here again the standard texts became a problem, as there is not a perfect overlap between important methods for general physics and important methods for optics. For example, group theory is not commonly used among most optics researchers, and Fourier transforms, essential to the optics researcher, are not used as much by the rest of the general physics community. Teaching to an optics crowd would require that the emphasis on material be retasked. It was in view of these various