

D-branes represent a key theoretical tool in the understanding of strongly coupled superstring theory and M-theory. They have led to many striking discoveries, including the precise microphysics underlying the thermodynamic behaviour of certain black holes, and remarkable holographic dualities between large- N gauge theories and gravity. This book provides a self-contained introduction to the technology of D-branes, presenting their development in a pedagogical manner. The introductory material is developed by first starting with the main features of string theory needed to get rapidly to grips with D-branes. Many advanced applications are covered, with discussions of open problems which could form the basis for other avenues of research. Suitable as a textbook in graduate courses on modern string theory and theoretical particle physics, it will also be an indispensable reference for seasoned practitioners. First published in 2003, this title has been reissued as an Open Access publication on Cambridge Core.

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