Einstein's General Theory of Relativity leads to two remarkable predictions: first, that the ultimate destiny of many massive stars is to undergo gravitational collapse and to disappear from view, leaving behind a 'black hole' in space; and secondly, that there will exist singularities in space-time itself. These singularities are places where space-time begins or ends, and the presently known laws of physics break down. They will occur inside black holes, and in the past are what might be construed as the beginning of the universe.

To show how these predictions arise, the authors discuss the General Theory of Relativity in the large. Starting with a precise formulation of the theory and an account of the necessary background of differential geometry, the significance of space-time curvature is discussed and the global properties of a number of exact solutions of Einstein's field equations are examined. The theory of the causal structure of a general spacetime is developed, and is used to study black holes and to prove a number of theorems establishing the inevitability of singularities under certain conditions. These conditions are shown to be satisfied in the vicinity of stars of more than twice the solar mass near the endpoint of their nuclear evolution, and in a time-reversed sense for the universe as a whole. In the first case, the singularity is inside a black hole, and in the second case, it is the initial singularity in our past. A discussion of the Cauchy problem for General Relativity is also included in the book.

The comments of the reviewers speak for themselves:

'The book is a masterpiece, written by sure hands.'

Science

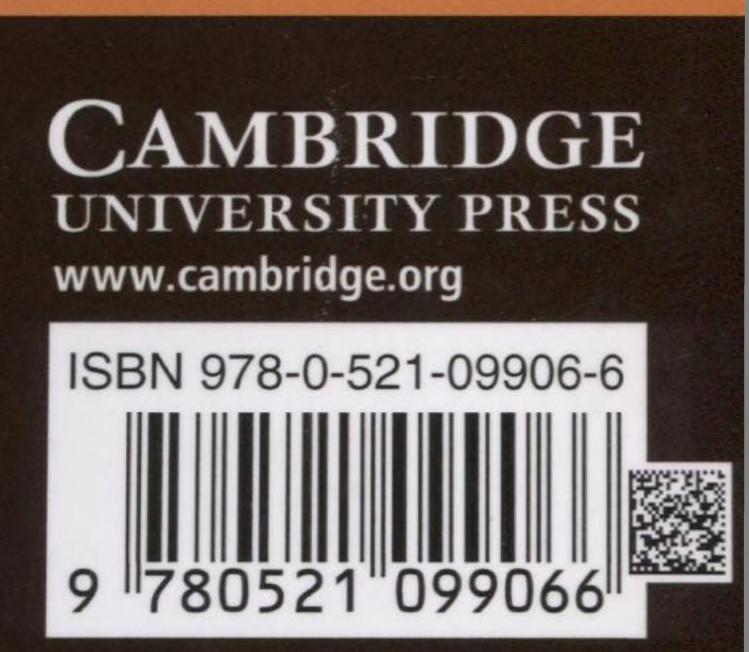
'Certain to rank as an outstanding classic of the current advance of relativity theory.'

Contemporary Physics

'La parution de ce livre est un événement important. . .'
La Recherche

Professor S.W. Hawking, FRS, is Lucasian Professor of Mathematics in the University of Cambridge

Professor Ellis is Professor of Applied Mathematics, University of Cape Town



Pre	face	page xi
1	The role of gravity	1
2	Differential geometry	
2.1	Manifolds	11
2.2	Vectors and tensors	15
2.3	Maps of manifolds	22
	Exterior differentiation and the Lie derivative	24
2.5	Covariant differentiation and the curvature tensor	30
2.6	The metric	36
2.7	Hypersurfaces	44
2.8	The volume element and Gauss' theorem	47
	Fibre bundles	50
3	General Relativity	56
3.1	The space-time manifold	56
3.2	The matter fields	59
3.3	Lagrangian formulation	64
3.4	The field equations	71
4	The physical significance of curvature	78
4.1	Timelike curves	78
4.2	Null curves	86
4.3	Energy conditions	88
4.4	Conjugate points	96
4.5	Variation of arc-length	102
5	Exact solutions	117
5.1	Minkowski space-time	118
5.2	De Sitter and anti-de Sitter space-times	124
5.3	Robertson-Walker spaces	134
5.4	Spatially homogeneous cosmological models	142
	[vii]	

5.5	The Schwarzschild and Reissner-Nordström	
	solutions	page 149
5.6	The Kerr solution	161
5.7	Gödel's universe	168
5.8	Taub-NUT space	170
5.9	Further exact solutions	178
6	Causal structure	180
6.1	Orientability	181
6.2	Causal curves	182
6.3	Achronal boundaries	186
6.4	Causality conditions	189
6.5	Cauchy developments	201
6.6	Global hyperbolicity	206
6.7	The existence of geodesics	213
6.8	The causal boundary of space-time	217
6.9	Asymptotically simple spaces	221
7	The Cauchy problem in General Relativity	226
7.1	The nature of the problem	227
7.2	The reduced Einstein equations	228
7.3	The initial data	231
7.4	Second order hyperbolic equations	233
7.5	The existence and uniqueness of developments fo	r
	the empty space Einstein equations	244
7.6	The maximal development and stability	249
7.7	The Einstein equations with matter	254
8	Space-time singularities	256
8.1	The definition of singularities	256
8.2	Singularity theorems	261
8.3	The description of singularities	276
8.4	The character of the singularities	284
8.5	Imprisoned incompleteness	289
9	Gravitational collapse and black holes	299
9.1	Stellar collapse	299
9.2	Black holes	308
9.3	The final state of black holes	323

CONTENTS

10	The initial singularity in the universe	page	348
10.1	The expansion of the universe		348
10.2	The nature and implications of singularities		359
	Appendix A:		
	Translation of an essay by P. S. Laplace		365
	Appendix B:		
	Spherically symmetric solutions and Birkhoff's		
	theorem		369
References			373
Notation			381
Index			385