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transmission over the channel. The "object" could be a sequence of binary digits, as in digital computer applications, or a continuous waveform, as in radio communication. The channel is the medium over which the coded message is transmitted. The decoder operates on the output of the channel and attempts to extract the original message for delivery to the destination. In general, this cannot be done with complete reliability because of the effect of "noise," which is a general term for anything which tends to produce errors in transmission.

Information theory is an attempt to construct a mathematical model for each of the blocks of Fig. 1.1.1. We shall not write out design formulas for a communication system; nevertheless, we shall go into considerable detail concerning the theory of the encoding and decoding operations.

It is possible to make a case for the statement that information theory is essentially the study of one theorem, the so-called "fundamental theorem of information theory," which states that "it is possible to transmit information through a noisy channel at any rate less than channel capacity with an arbitrarily small probability of error." The meaning of the various terms "information," "channel," "noise," "rate," and "capacity" will be defined in later chapters. At this point, we shall only try to give an intuitive idea of the content of the fundamental theorem.



Fig. 1.1.1. Communication system.