

This book presents in a detailed and self-contained way a new and important density result in the analysis of fractional partial differential equations, while also covering several fundamental facts about space- and time-fractional equations.

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ISBN 978-3-11-066069-2

ISSN 0179-0986

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