

Lecture Notes in Physics

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Introduction to Muon Spin Spectroscopy

Applications to Solid State and Material Sciences

This textbook serves as a comprehensive introduction to muon spin spectroscopy (μ SR), offering a detailed exploration of how polarized positive muons can be employed as local probes to investigate material properties at the microscopic level. It provides a self-contained tutorial that begins by explaining the extraction of physical information from a μ SR experiment and then proceeds to present illustrative examples in the fields of condensed matter physics, materials science, and nanoscience.

The book focuses on major applications of μ SR, including the study of magnetism, superconductivity, and semiconducting materials in both bulk and thin film samples. In addition, two chapters delve into the applications of negative muons, emphasizing their role in elemental materials analysis and introducing fundamental particle physics aspects of muon science. Supplementary material, conveniently summarized in several appendices, covers essential basic concepts. For further exploration, an extensive list of references is provided, enabling readers to deepen their knowledge in specific areas.

To facilitate understanding and mastery of the subject, the textbook offers exercises and solutions. It caters to advanced undergraduate, graduate and PhD level students, researchers who intend to utilize the μ SR technique or seek a comprehensive understanding of μ SR results for their research, as well as to established practitioners.

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