

“In a difficult 1968 paper Dyson and Lenard succeeded in proving the ‘Stability of Matter’ in quantum mechanics. In 1975 a much simpler proof was developed by Thirring and me with a new, multi-function, Sobolev-like inequality, as well as a bound on the negative spectrum of Schrödinger operators. These and other bounds have become an important and useful branch of functional analysis and differential equations generally and quantum mechanics in particular. This book, written by three of the leading contributors to the area, carefully lays out the entire subject in a highly readable, yet complete, description of these inequalities. They also give gently, yet thoroughly, all the necessary spectral theory and Sobolev theory background that a beginning student might need.”

**Professor Elliott Lieb, Princeton University**

“In 1975, Lieb and Thirring proved a remarkable bound of the sum of the negative eigenvalues of a Schrödinger operator in three dimensions in terms of the  $L^{5/2}$ -norm of the potential and used it in their proof of the stability of matter. Shortly thereafter they realized it was a case of a lovely set of inequalities which generalize Sobolev inequalities and have come to be called Lieb–Thirring bounds. This has spawned an industry with literally hundreds of papers on extensions, generalizations and optimal constants. It is wonderful to have the literature presented and synthesized by three experts who begin by giving the background necessary for this book to be useful not only to specialists but to the novice wishing to understand a deep chapter in mathematical analysis.”

**Professor Barry Simon, California Institute of Technology**

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