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Foreword

There are many excellent books that treat the mathematical and physical foundations of quantum mechanics. The question is: why another one? Approaching physics subjects usually progresses like this: a young individual or a curious adult is interested in physics and tries to understand it better by reading a popular science book. By reading it, he/she gets a gist of the subjects – for example, black holes or lasers – but gains little clarity. The book sparks many questions. The most common questions are “why?” or “how do they know?” and so on. There is therefore a moment when he/she is not anymore happy with just a qualitative description of nature but wants to know the inner to understand everything a bit more. He/she starts to read books that are not only about the way scientists make predictions. Given some amount of knowledge about a physical system, he/she wants to learn how to calculate how it will behave in the future. For example, given position and velocity of a point-like particle at time t_0 , he/she wants to know where it will be at a later time $t_1 > t_0$.

If the individual in question decides to understand physics systematically, he/she usually attends a University study program where somebody gives lectures and recommends books. Some of the most important initial concepts are how to measure things and how to express the measurements in a systematic way. The next step is to try to see if there are patterns in the numbers when we try to change something. Suppose the point-like mass is in the gravitational field of the Earth, and we drop it from some height and we measure the position of the mass with time. If we plot the position versus time, we see a beautiful curve emerging from the data: a parabola. It seems therefore that we can use nature to describe the physical world. Nature seems to obey laws that can be expressed in mathematical form. In fact, we can fit the data collected for the falling object with a simple expression $x(t) = x_0 + v_0 t - \frac{1}{2} g t^2$, which allows us to make predictions, i.e. we can predict where the object will accelerate if we drop it from higher and higher altitudes... or not? But the magic seems to be in the exponent 2 in the previous expression and in the constant g . Why 2? Is it exactly 2 or perhaps 1.99998 or 2.00001? Is the exponent 2 due to gravity? If yes, why and how? The regularity or patterns in the data suggests that there is a mathematical relation that governs the data. This is the first step towards a physical law. A physical law is a recipe that allows anybody to make predictions given some knowledge of nature. Given two systems in exactly the same state, they will evolve with time in exactly the same way. This is more or less classical mechanics with its infinite precision in the knowledge of physical quantities and absolute certainty in the time evolution of a system. Classical mechanics is the common sense of everyday life of human beings when speed is not too high, and distances are not too big or too small.

When nature started to be probed at very small scales – for example, atomic lengths – new phenomena emerged that could not be described with the mathematics used by classical mechanics. New equations and new tools needed to be employed to try to make predictions. However, even the act of measurements needed to be