

This series is devoted to thorough yet reasonably concise treatments of topics in any branch of mathematics. Typically, a Tract takes up a single thread in a wide subject and follows its ramifications, throwing light on various of its aspects. Tracts are expected to be rigorous, definitive, and of lasting value to the mathematicians working in the relevant disciplines. Exercises can be included to illustrate techniques, summarize past work, and enhance the book's value as a seminar text. All volumes are properly edited and type-set and are published, initially at least, in hardcover.

Ákos Seress is a Professor of Mathematics at The Ohio State University.

CAMBRIDGE TRACTS IN MATHEMATICS

GENERAL EDITORS

B. BOLLOBAS, W. FULTON, A. KATOK, F. KIRWAN, P. SARNAK

A complete list of the series can be found at <http://www.cambridge.org>

Recent titles include the following:

115. An Introduction to Hp Spaces (2nd ed.) .By P.KOOSIS
116. Matrices of Sign-Solvable Linear Systems.By R.A.BRUALDI and B.L.SHADER
117. Generalized Topological Degree and Semilinear Equations.By W .PETRYSHYN
118. Sets of Multiples.By R.R.HALL
119. Continuum Percolation.By R.MEESTER and R.ROY
120. Function Spaces, Entropy Numbers and Differential Operators.By D.E.EDMUNDS and H.TRIEBEL
121. Lévy Processes.By J.BERTOIN
122. Duality in Analytic Number Theory.By P.D.T.A.ELLIOTT
123. Ends of Complexes.By A.RANICKI and B.HUGHES
124. 3-Transposition Groups.By M.ASCHBACHER
125. The Hardy-Littlewood Method (2nd ed.) .By R.C.VAUGHAN
126. Dynamical Systems and Semisimple Groups.By R.FERES
127. Spectral Theory of the Riemann Zeta-Function.By Y.MOTOHASHI
128. Introduction to Maximum Principles.By L.E.FRAENKEL
129. Gaussian Hilbert Spaces.By S.JANSON
130. Automorphic Forms on $SL_2(\mathbb{R})$.By A.BOREL
131. Bipartite Graphs and Their Applications.By A.ASRATIAN, T.DENLEY, and R.HÄGGKVIST
132. Mixed Hodge Structures and Singularities.By V.KULIKOV
133. Multiplicities and Chern Classes in Local Algebra.By P.ROBERTS
134. Birational Geometry of Algebraic Varieties.By J.KOLLÁR and S.MORI
135. Solutions.By T.MIWA, M.JIMBO, and E.DATE
136. Character Sums with Exponential Functions and Their Applications.By S.KONYAGIN and I.SHPARLINSKI
137. Metric Diophantine Approximation on Manifolds.By V.I.BERNIK and M.M.DODSON
138. Random Walks on Infinite Graphs and Groups.By WOLFGANG WOESS
139. Measure-Preserving Homeomorphisms.By STEVE ALPERN and V.S.PRASAD
140. Derivation and Integration.By WASHEK F.PFEFFER
141. Fixed Point Theory and Applications.By RAVI P.AGARWAL, MARIA MEEHAN, and DONAL O'REGAN
142. Harmonic Maps between Riemannian Polyhedra.By JAMES EELLS and BENT FUGLEDE
143. Analysis on Fractals.By JUN KIGAMI
144. Torsors and Rational Points.By A.SKOROBOGATOV
145. Isoperimetric Inequalities.By ISAAC CHAVEL
146. Restricted Orbit Equivalence for Actions of Discrete Amenable Groups.By J.KAMMEYER and D.RUDOLPH
147. Floer Homology Groups in Yang-Mills Theory.By S.K.DONALDSON
149. Cohomology of Vector Bundles and Syzygies.By JERZY WEYMAN
150. Harmonic Maps, Conservation Laws and Moving Frames.By FRÉDÉRIC HÉLEIN
151. Frobenius Manifolds and Moduli Spaces for Singularities.By CLAUS HERTLING

CAMBRIDGE
UNIVERSITY PRESS
www.cambridge.org

ISBN 978-0-521-66103-4



9 780521 661034 >



1	Introduction	<i>page</i> 1
1.1	A List of Algorithms	4
1.2	Notation and Terminology	6
1.2.1	Groups	7
1.2.2	Permutation Groups	9
1.2.3	Algorithmic Concepts	10
1.2.4	Graphs	11
1.3	Classification of Randomized Algorithms	12
2	Black-Box Groups	16
2.1	Closure Algorithms	18
2.1.1	Orbit Computations	18
2.1.2	Closure of Algebraic Structures	23
2.2	Random Elements of Black-Box Groups	24
2.3	Random Subproducts	30
2.3.1	Definition and Basic Properties	30
2.3.2	Reducing the Number of Generators	33
2.3.3	Closure Algorithms without Membership Testing	37
2.3.4	Derived and Lower Central Series	38
2.4	Random Prefixes	40
2.4.1	Definition and Basic Properties	40
2.4.2	Applications	44
3	Permutation Groups: A Complexity Overview	48
3.1	Polynomial-Time Algorithms	48
3.2	Nearly Linear-Time Algorithms	51
3.3	Non-Polynomial-Time Methods	52

4	Bases and Strong Generating Sets	55
4.1	Basic Definitions	55
4.2	The Schreier–Sims Algorithm	57
4.3	The Power of Randomization	62
4.4	Shallow Schreier Trees	64
4.5	Strong Generators in Nearly Linear Time	70
4.5.1	Implementation	75
5	Further Low-Level Algorithms	79
5.1	Consequences of the Schreier–Sims Method	79
5.1.1	Pointwise Stabilizers	79
5.1.2	Homomorphisms	80
5.1.3	Transitive Constituent and Block Homomorphisms	81
5.1.4	Closures and Normal Closures	83
5.2	Working with Base Images	84
5.3	Permutation Groups as Black-Box Groups	93
5.4	Base Change	97
5.5	Blocks of Imprimitivity	100
5.5.1	Blocks in Nearly Linear Time	101
5.5.2	<i>The Smallest Block Containing a Given Subset</i>	<i>107</i>
5.5.3	Structure Forests	111
6	A Library of Nearly Linear-Time Algorithms	114
6.1	A Special Case of Group Intersection and Applications	115
6.1.1	Intersection with a Normal Closure	115
6.1.2	Centralizer in the Symmetric Group	117
6.1.3	The Center	120
6.1.4	Centralizer of a Normal Subgroup	120
6.1.5	Core of a Subnormal Subgroup	124
6.2	Composition Series	125
6.2.1	Reduction to the Primitive Case	126
6.2.2	The O’Nan–Scott Theorem	129
6.2.3	Normal Subgroups with Nontrivial Centralizer	133
6.2.4	<i>Groups with a Unique Nonabelian Minimal Normal Subgroup</i>	<i>139</i>
6.2.5	Implementation	146
6.2.6	An Elementary Version	149
6.2.7	Chief Series	155
6.3	Quotients with Small Permutation Degree	156
6.3.1	Solvable Radical and p -Core	157

7	Solvable Permutation Groups	162
7.1	Strong Generators in Solvable Groups	162
7.2	Power-Conjugate Presentations	165
7.3	Working with Elementary Abelian Layers	166
7.3.1	Sylow Subgroups	167
7.3.2	Conjugacy Classes in Solvable Groups	172
7.4	Two Algorithms for Nilpotent Groups	175
7.4.1	A Fast Nilpotency Test	176
7.4.2	The Upper Central Series in Nilpotent Groups	179
8	Strong Generating Tests	183
8.1	The Schreier–Todd–Coxeter–Sims Procedure	184
8.1.1	Coset Enumeration	184
8.1.2	Leon’s Algorithm	186
8.2	Sims’s Verify Routine	188
8.3	Toward Strong Generators by a Las Vegas Algorithm	191
8.4	A Short Presentation	197
9	Backtrack Methods	201
9.1	Traditional Backtrack	202
9.1.1	Pruning the Search Tree: Problem-Independent Methods	203
9.1.2	Pruning the Search Tree: Problem-Dependent Methods	205
9.2	The Partition Method	207
9.3	Normalizers	211
9.4	Conjugacy Classes	214
10	Large-Base Groups	218
10.1	Labeled Branchings	218
10.1.1	Construction	222
10.2	Alternating and Symmetric Groups	225
10.2.1	Number Theoretic and Probabilistic Estimates	228
10.2.2	Constructive Recognition: Finding the New Generators	235
10.2.3	Constructive Recognition: The Homomorphism λ	239
10.2.4	Constructive Recognition: The Case of Giants	244
10.3	A Randomized Strong Generator Construction	246
	Bibliography	254
	Index	262