

# Linear Algebra and Geometry

A I Kostrikin and Yu I Manin

Steklov Institute of Mathematics, USSR Academy of Sciences, Moscow, USSR

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This advanced textbook on linear algebra and geometry covers a wide range of classical and modern topics. Differing from most existing textbooks in approach, the work illustrates the many-sided applications and connections of linear algebra with functional analysis, quantum mechanics, and algebraic and differential geometry. The subjects covered in some detail include normed linear spaces, functions of linear operators, the basic structures of quantum mechanics and an introduction to linear programming. Also discussed are Kahler's metric, the theory of Hilbert polynomials, and projective and affine geometries. Unusual in its extensive use of applications in physics to clarify each topic, this comprehensive volume will be of particular interest to advanced undergraduates and graduates in mathematics and physics, and to lecturers in linear and multilinear algebra, linear programming and quantum mechanics.

## About the authors

**Aleksei I Kostrikin** is currently a Corresponding Member of the USSR Academy of Sciences and holds the Chair in Algebra at Moscow State University. A winner of the USSR Award in Mathematics in 1968, Professor Kostrikin's main research interests are Lie algebras and finite groups.

**Yuri I Manin** is currently Senior Research Staff Member at the Steklov Institute of the Academy of Sciences of the USSR and Professor of Algebra at Moscow State University. Professor Manin has been awarded the Lenin Prize for work in algebraic geometry and the Brouwer Gold Medal for work in number theory. His research interests also include differential equations and quantum field theory.

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# Contents

Preface	vii
Bibliography	ix
<b>CHAPTER 1</b> <b>Linear Spaces and Linear Mappings</b>	<b>1</b>
1    Linear Spaces	1
2    Basis and Dimension	8
3    Linear Mappings	16
4    Matrices	22
5    Subspaces and Direct Sums	34
6    Quotient Spaces	43
7    Duality	48
8    The Structure of a Linear Mapping	52
9    The Jordan Normal Form	58
10   Normed Linear Spaces	66
11   Functions of Linear Operators	72
12   Complexification and Decomplexification	75
13   The Language of Categories	81
14   The Categorical Properties of Linear Spaces	87
 <b>CHAPTER 2</b> <b>Geometry of Spaces with an Inner Product</b>	 <b>92</b>
1    On Geometry	92
2    Inner Products	94
3    Classification Theorems	101
4    The Orthogonalization Algorithm and Orthogonal Polynomials	109
5    Euclidean Spaces	117
6    Unitary Spaces	127
7    Orthogonal and Unitary Operators	134
8    Self-Adjoint Operators	138
9    Self-Adjoint Operators in Quantum Mechanics	148
10   The Geometry of Quadratic Forms and the Eigenvalues of Self-Adjoint Operators	156
11   Three-Dimensional Euclidean Space	164
12   Minkowski Space	173
13   Symplectic Space	182



14	Witt's Theorem and Witt's Group	187
15	Clifford Algebras	190
<b>CHAPTER 3 Affine and Projective Geometry</b>		<b>195</b>
1	Affine Spaces, Affine Mappings, and Affine Coordinates	195
2	Affine Groups	203
3	Affine Subspaces	207
4	Convex Polyhedra and Linear Programming	215
5	Affine Quadratic Functions and Quadrics	218
6	Projective Spaces	222
7	Projective Duality and Projective Quadrics	228
8	Projective Groups and Projections	233
9	Desargues' and Pappus' Configurations and Classical Projective Geometry	242
10	The Kahler Metric	247
11	Algebraic Varieties and Hilbert Polynomials	249
<b>CHAPTER 4 Multilinear Algebra</b>		<b>258</b>
1	Tensor Products of Linear Spaces	258
2	Canonical Isomorphisms and Linear Mappings of Tensor Products	263
3	The Tensor Algebra of a Linear Space	269
4	Classical Notation	271
5	Symmetric Tensors	276
6	Skew-Symmetric Tensors and the Exterior Algebra of a Linear Space	279
7	Exterior Forms	290
8	Tensor Fields	293
9	Tensor Products in Quantum Mechanics	297
<b>Index</b>		<b>303</b>