Actuaries must pass exams, but more than that: they must put knowledge into practice. This coherent book supports the Society of Actuaries' short-term actuarial mathematics syllabus while emphasizing the concepts and practical application of nonlife actuarial models. A class-tested textbook for undergraduate courses in actuarial science, it is also ideal for those approaching their professional exams. Key topics covered include loss modeling, risk and ruin theory, credibility theory and applications and empirical implementation of loss models.

Revised and updated to reflect curriculum changes, this second edition includes two brand-new chapters on loss reserving and ratemaking. R replaces Excel as the computation tool used throughout – the featured R code is available on the book's webpage, as are lecture slides. Numerous examples and exercises are provided, with many questions adapted from past Society of Actuaries exams.

"This book covers the body of knowledge required by the Society of Actuaries (SOA) for its Fundamentals of Actuarial Mathematics Exam (Sections for short-term coverages) starting from 2023. I believe that it is an ideal textbook for a semester course on the introduction of short-term actuarial mathematics. It is also useful for self-study candidates preparing for the actuarial professional exams."

Wai-Sum Chan, PhD, FSA, HonFIA, CERA Dean, *School of Decision Sciences*, *The Hang Seng University of Hong Kong*

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