

Discrete Mathematics with Graph Theory

This book is designed to meet the requirement of undergraduate and postgraduate students pursuing computer science, information technology, mathematical science, and physical science course. No formal prerequisites are needed to understand the text matter except a very reasonable background in college algebra. The text contains in-depth coverage of all major topics proposed by professional institutions and universities for a discrete mathematics course. It emphasizes on problem-solving techniques, pattern recognition, conjecturing, induction, applications of varying nature, proof technique, algorithmic development, algorithm correctness, and numeric computations. A sufficient amount of theory is included for those who enjoy the beauty in development of the subject and a wealth of applications as well as for those who enjoy the power of problem-solving techniques.

Biographical sketches of nearly 25 mathematicians and computer scientists who have played a significant role in the development of the field are threaded into the text to provide a human dimension and attach a human face to major discoveries. Each section of the book contains a generous selection of carefully tailored examples to classify and illuminate various concepts and facts. Theorems are backbone of mathematics. Consequently, this book contains the various proof techniques, explained and illustrated in details. Most of the concepts, definitions, and theorems in the book are illustrated with appropriate examples. Proofs shed additional light on the topic and enable students to sharpen their problem-solving skills. Each chapter ends with a summary of important vocabulary, formulae, properties developed in the chapter, and list of selected references for further exploration and enrichment.

ISBN 978-3-031-21323-6



0. PRELIMINARIES	1–14
0.1 Numbers	1
0.2 Euclid's Algorithm	3
0.3 Fundamental Theorem of Arithmetic	4
0.4 Euclid's Theorem	6
0.5 Congruence Modulo m	6
0.6 Chinese Remainder Theorem	7
0.7 Fermat's and Euler's Theorems	9
0.8 Exponents and Logarithms	10
0.9 Sums	11
0.10 Mapping	12
<i>Suggested Readings</i>	14
1. THE LANGUAGE OF SETS	15–66
1.1 Introduction	15
1.2 Elements and Notations of Sets	16
1.3 Construction of Sets	17
1.4 Types of Sets	19
1.5 Set Operations	25
1.5.1 Intersection of Sets	25
1.5.2 Union of Sets	26
1.5.3 Disjoint Set (Mutually Exclusive)	27
1.5.4 Ordinary Difference of Sets ($A - B$)	27
1.5.5 Complementation of Sets	27

1.5.6	Universal Set and its Complement	27
1.5.7	Symmetric Difference (Boolean Sum)	28
1.6	Venn Diagrams	28
1.7	Some Basic Results	32
1.8	Properties of Set Operations	34
1.8.1	Properties of Intersection on Sets	34
1.8.2	Properties of Union of Sets	35
1.8.3	Number of Elements in a Union of two or more Sets	39
1.9	De-Morgan's Laws	40
1.10	General form of Principle of Inclusion and Exclusion	44
1.11	Laws of Sets	63
	<i>Summary</i>	63
	<i>Suggested Readings</i>	65

2. BASIC COMBINATORICS 67–114

2.1	Introduction	67
2.2	Basic Counting Principles	68
2.2.1	The Principle of Disjunctive Counting (Sum Rule)	68
2.2.2	The Principle of Sequential Counting (Product Rule)	69
2.3	Factorial	71
2.4	Permutation and Combination	73
2.4.1	Cyclic Permutation	76
2.4.2	Pascal's Identity	76
2.4.3	Vandermonde's Identity	77
2.4.4	Pigeonhole Principle	78
2.4.5	Inclusion–Exclusion Principle	79
2.5	The Binomial Theorem	93
2.6	n th Catalan Number	95
2.7	Principle of Mathematical Induction (P.M.I)	96
2.8	Recurrence Relations	99
	<i>Summary</i>	110
	<i>Suggested Readings</i>	113

3. MATHEMATICAL LOGIC

115–180

3.1	Introduction	115
3.2	Propositions (Statements)	117
3.3	Connectives	117
3.3.1	Negation	118
3.3.2	Conjunction	119
3.3.3	Disjunction	119
3.3.4	Conditional	120
3.3.5	Biconditional	120
3.4	Equivalence of Formulae	121
3.5	Well-Formed Formulae (WFF)	122
3.6	Tautologies	122
3.7	Principle of Duality	123
3.8	Two State Devices	128
3.9	The Relay-Switching Devices	129
3.10	Logic Gates and Modules	130
3.10.1	OR, AND and NOT Gates	130
3.10.2	Two-Level Networks	132
3.10.3	NOR and NAND Gates	132
3.11	Normal Forms (Decision Problems)	141
3.11.1	Disjunctive Normal Form (DNF)	141
3.11.2	Conjunctive Normal Form (CNF)	145
3.11.3	Principal Disjunctive Normal Form (PDNF)	146
3.11.4	Principal Conjunctive Normal Forms (PCNF)	148
3.12	Rules of Inference	151
3.13	Automatic Proving System (Theorems)	152
3.14	The Predicate Calculus	164
3.14.1	Statement Functions, Variables and Quantifiers	166
3.14.2	Free and Bound Variables	166
3.14.3	Special Valid Formulae using Quantifiers	167
3.14.4	Theory of Inference for the Predicate Calculus	168
3.14.5	Formulae Involving More than one Quantifier	169
	<i>Summary</i>	175
	<i>Suggested Readings</i>	179

4. RELATIONS 181–236

4.1	Introduction	181
4.2	Product Sets	182
4.3	Partitions	182
4.4	Relations	183
4.5	Binary Relations in a Set	183
4.6	Domain and Range of a Relation	184
4.6.1	Number of Distinct Relation From set A to B	185
4.6.2	Solution sets and Graph of Relations	189
4.6.3	Relation as Sets of Ordered Pairs	190
4.7	The Matrix of a Relation and Digraphs	190
4.8	Paths in Relations and Digraphs	191
4.9	Boolean Matrices	194
4.9.1	Boolean Operations AND and OR	195
4.9.2	Joint and Meet	195
4.9.3	Boolean Product	195
4.9.4	Boolean Power of a Boolean Matrix	195
4.10	Adjacency Matrix of a Relation	198
4.11	Gray Code	198
4.12	Properties of Relations	200
4.12.1	Reflexive and Irreflexive Relations	201
4.12.2	Symmetric, Asymmetric and Antisymmetric Relations	201
4.12.3	Transitive Relation	202
4.13	Equivalence Relations	205
4.14	Closure of Relations	207
4.15	Manipulation and Composition of Relations	208
4.16	Warshall's Algorithm	216
4.17	Partial Order Relation	225
4.17.1	Totally Ordered Set	226
4.17.2	Lexicographic Order	226
4.17.3	Hasse Diagrams	228
	<i>Summary</i>	230
	<i>Suggested Readings</i>	235

5. FUNCTIONS 237–270

5.1	Introduction	238
5.1.1	Sum and Product of Functions	239
5.2	Special Types of Functions	242
5.2.1	Polynomial Function	244
5.2.2	Exponential and Logarithmic Function	244
5.2.3	Floor and Ceiling Functions	245
5.2.4	Transcendental Function	247
5.2.5	Identity Function	247
5.2.6	Integer Value and Absolute Value Functions	247
5.2.7	Remainder Function	248
5.3	Composition of Functions	248
5.4	Inverse of a Function	250
5.5	Hashing Functions	256
5.6	Countable and Uncountable Sets	257
5.7	Characteristic Function of a Set	259
5.8	Permutation Function	261
5.9	Growth of Functions	262
5.10	The Relation Θ	262
	<i>Summary</i>	267
	<i>Suggested Readings</i>	269

6. LATTICE THEORY 271–304

6.1	Introduction	271
6.2	Partial Ordered Sets	272
6.2.1	Some Important Terms	273
6.2.2	Diagramatical Representation of a Poset (Hasse Diagram)	275
6.2.3	Isomorphism	276
6.2.4	Duality	278
6.2.5	Product of two Posets	280
6.3	Lattices as Posets	282
6.3.1	Some Properties of Lattices	283
6.3.2	Lattices as Algebraic Systems	284

6.3.3	Complete Lattice	290
6.3.4	Bounded Lattice	290
6.3.5	Sublattices	291
6.3.6	Ideals of Lattices	291
6.4	Modular and Distributive Lattices	292
	<i>Summary</i>	302
	<i>Suggested Readings</i>	304

7. BOOLEAN ALGEBRAS AND APPLICATIONS 305–354

7.1	Introduction	305
7.2	Boolean Algebra (Analytic Approach)	306
7.2.1	Sub-Boolean Algebra	308
7.2.2	Boolean Homomorphism	309
7.3	Boolean Functions	318
7.3.1	Equality of Boolean Expressions	319
7.3.2	Minterms and Maxterms	319
7.3.3	Functional Completeness	320
7.3.4	NAND and NOR	320
7.4	Combinatorial Circuits (Synthesis of Circuits)	326
7.4.1	Half-Adder and Full-Adder	326
7.4.2	Equivalent Combinatorial Circuits	328
7.5	Karnaugh Map	331
7.5.1	Don't Care Conditions	334
7.5.2	Minimization Process	335
7.6	Finite State Machines	344
	<i>Summary</i>	347
	<i>Suggested Readings</i>	352

8. FUZZY ALGEBRA 355–392

8.1	Introduction	355
8.2	Crisp Sets and Fuzzy Sets	357
8.3	Some Useful Definitions	360
8.4	Operations of Fuzzy Sets	362
8.5	Interval-Valued Fuzzy Sets (I-V Fuzzy Sets)	367
8.5.1	Union and Intersection of two I-V Fuzzy Sets	368

8.6	Fuzzy Relations	369
8.6	Fuzzy Measures	373
8.7.1	Belief and Plausibility Measures	373
8.7.2	Probability Measure	374
8.7.3	Uncertainty and Measures of Fuzziness	374
8.7.4	Uncertainty and Information	375
8.8	Applications of Fuzzy Algebras	376
8.8.1	Natural, Life and Social Sciences	376
8.8.2	Engineering	378
8.8.3	Medical Sciences	381
8.8.4	Management Sciences and Decision Making Process	382
8.8.5	Computer Science	383
8.9	Uniqueness of Uncertainty Measures	384
8.9.1	Shannon's Entropy	384
8.9.2	U-uncertainty	386
8.9.3	Uniqueness of the U-uncertainty for Two-Value Possibility Distributions	388
	<i>Summary</i>	389
	<i>Suggested Readings</i>	390

9. FORMAL LANGUAGES AND AUTOMATA THEORY 393–428

9.1	Introduction	393
9.2	Formal Languages	396
9.2.1	Equality of Words	397
9.2.2	Concatenation of Languages	398
9.2.3	Kleene Closure	399
9.3	Grammars	403
9.3.1	Phase-structure Grammar	406
9.3.2	Derivations of Grammar	406
9.3.3	Backus-Normal Form (BNF) or Backus Naur Form	407
9.3.4	Chomsky Grammar	410
9.3.5	Ambiguous Grammar	411