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A Course in Combinatorics and Graphs

This compact textbook consists of lecture notes given as a fourth-year undergraduate course of the mathematics degree at the Universitat Politècnica de Catalunya, including topics in enumerative combinatorics, finite geometry, and graph theory. This text covers a single-semester course and is aimed at advanced undergraduates and masters-level students. Each chapter is intended to be covered in 6-8 hours of classes, which includes time to solve the exercises. The text is also ideally suited for independent study. Some hints are given to help solve the exercises and if the exercise has a numerical solution, then this is given. The material covered allows the reader with a rudimentary knowledge of discrete mathematics to acquire an advanced level on all aspects of combinatorics, from enumeration, through finite geometries to graph theory.

The intended audience of this book assumes a mathematical background of third-year students in mathematics, allowing for a swifter use of mathematical tools in analysis, algebra, and other topics, as these tools are routinely incorporated in contemporary combinatorics. Some chapters take on more modern approaches such as Chapters 1, 2, and 9. The authors have also taken particular care in looking for clear concise proofs of well-known results matching the mathematical maturity of the intended audience.

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