

Association for Women in Mathematics Series

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Research Directions in Number Theory

Women in Numbers V

This is the fifth proceedings volume published under the Women in Numbers umbrella. The WIN workshops and their proceedings volumes are part of the WIN network, aimed at highlighting the research of women and gender minorities in number theory as well as increasing their participation and boosting their potential collaborations in number theory and related fields.

The volume contains research articles in the mathematical area of number theory, written by teams of scholars at all levels in the field. More information about the network, its goals and purpose, past and future conferences, and past proceedings volumes, can be found on the WIN website.

This volume contains research outcomes and results produced by the collaborative research groups created under the Women in Numbers V workshop, the 5th in its series. The actual workshop was to take place in 2020 at the Banff International Research Station in Banff, Canada, but could not take place onsite due to COVID. The associated research groups, each consisting of 1-2 leaders and 2-4 junior researchers, were formed nevertheless and their collaborations went ahead in purely virtual form, as well as other papers by author teams for which at least 50% of the authors identify as women or gender minorities. These contributions include original research and survey articles in a wide variety of subareas within number theory. The former present new cutting-edge research that will be of interest to experts in the field, to the benefit of their own research. The survey articles serve as an accessible introduction for graduate students and other readers to areas of number theory that may be outside their area of expertise.

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