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Knots and Primes

An Introduction to Arithmetic Topology, Second Edition

This book provides a foundation for arithmetic topology, a new branch of mathematics that investigates the analogies between the topology of knots, 3-manifolds, and the arithmetic of number fields. Arithmetic topology is now becoming a powerful guiding principle and driving force to obtain parallel results and new insights between 3-dimensional geometry and number theory.

After an informative introduction to Gauss' work, in which arithmetic topology originated, the text reviews a background from both topology and number theory. The analogy between knots in 3-manifolds and primes in number rings, the founding principle of the subject, is based on the étale topological interpretation of primes and number rings. On the basis of this principle, the text explores systematically intimate analogies and parallel results of various concepts and theories between 3-dimensional topology and number theory. The presentation of these analogies begins at an elementary level, gradually building to advanced theories in later chapters. Many results presented here are new and original.

References are clearly provided if necessary, and many examples and illustrations are included. Some useful problems are also given for future research. All these components make the book useful for graduate students and researchers in number theory, low dimensional topology, and geometry.

This second edition is a corrected and enlarged version of the original one. Misprints and mistakes in the first edition are corrected, references are updated, and some expositions are improved. Because of the remarkable developments in arithmetic topology after the publication of the first edition, the present edition includes two new chapters. One is concerned with idelic class field theory for 3-manifolds and number fields. The other deals with topological and arithmetic Dijkgraaf–Witten theory, which supports a new bridge between arithmetic topology and mathematical physics.

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