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## Zero and Pi

Symbols of Mathematical Spirit

The book, divided into two major parts, discusses the evolution of the concept and symbols of zero and the history of pi. Both the topics are discussed from the Neolithic Age to the nineteenth century. The book also clears the assumption that Johann Heinrich Lambert (AD 1761) only invented the irrationality of pi by crediting Lambert jointly with André Marie Legendre (AD 1794).

Part 1, consisting of six stages spread in six chapters, meets a challenge to the authors as eminent scholars of the history of mathematics have diverse opinions based on conjectures. This part primarily discusses how the symbol O, in the Vedic religious practices, considered a replica of the universe prescribed for meditation on the unknown Brahman (conceived of as the space supreme in the Upanishads), was later transcended to the symbol of an unknown quantity in mathematics along with a dot for zero in an arena of atheism. It also highlights how the zero notation and the decimal system of Indian numerals embellished with the algebraic thoughts of Brahmagupta passed on to China and Europe via Arabia. Topics in this part have traced the development from the origin to the final form as seen today after the western practice and try to put an end to the long-standing debate over history. Appendices contain the Sanskrit verses (transliterated with meanings into English) along with the essential mathematical deduction referred to in the body of the part to help the reader to have a better understanding.

Part 2 speaks of a novel idea of unveiling the nature of pi interwoven with threads of historical ups and downs in the world scenario. This part, containing five chapters, collects all available up-to-date data in every field of history to make the presentation complete in all respects. This part discusses the origin of the definition of pi as the rim of a wheel is thrice its diameter at the Indus Valley in the fourth millennium BC. This part also discusses the enlightenment of China in circle-squaring (classical method), Indian mathematics with astronomical knowledge along the Buddhist channel, and India's discovering circumference/diameter as a non-Euclidean number.

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