

A Compendium of **Musical Mathematics**

The purpose of this book is to provide a concise introduction to the mathematical theory of music, opening each chapter to the most recent research. Despite the complexity of some sections, the book can be read by a large audience. Many examples illustrate the concepts introduced. The book is divided into 9 chapters.

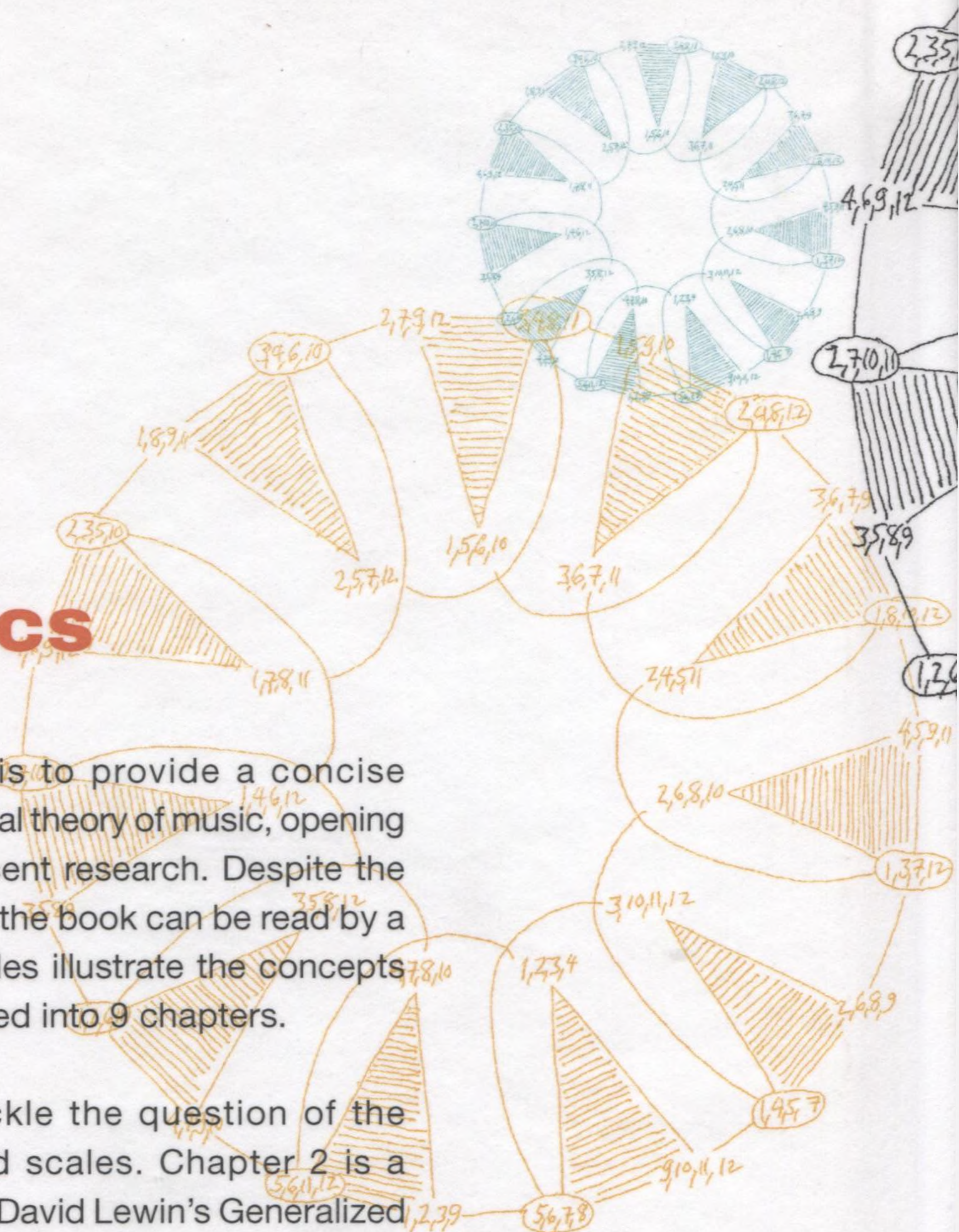
In the first chapter, we tackle the question of the classification of chords and scales. Chapter 2 is a mathematical presentation of David Lewin's Generalized Interval Systems. Chapter 3 offers a new theory of diatonicity in equal-tempered universes. Chapter 4 presents the Neo-Riemannian theories based on the work of David Lewin, Richard Cohn and Henry Klumpenhouwer. Chapter 5 is devoted to the application of word combinatorics to music. Chapter 6 studies the rhythmic canons and the tessellation of the line. Chapter 7 is devoted to serial knots. Chapter 8 presents combinatorial designs and their applications to music. The last chapter, chapter 9, is dedicated to the study of tuning systems.

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