

The aim of this graduate textbook is to provide a comprehensive advanced course in the theory of statistics covering those topics in estimation, testing, and large sample theory which a graduate student might typically need to learn as preparation for work on a Ph.D. An important strength of this book is that it provides a mathematically rigorous account of both classical and Bayesian inference in order to give readers a broad perspective. For example, the “uniformly most powerful” approach to testing is contrasted with available decision-theoretic approaches.

Commencing with chapters on probability models and the theory of sufficient statistics, the author covers decision theory, hypothesis testing, estimation, equivariance, large sample theory, hierarchical models, and, finally, sequential analysis. Every chapter concludes with exercises which range in difficulty from the easy to the challenging. As a result, this textbook provides an excellent course in modern theoretical statistics.

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*Sections and chapters marked with an asterisk may be skipped or covered out of order without interrupting the flow of ideas.

⁺Sections marked with a plus sign include results which rely on the theory of martingales. They may be skipped without interrupting the flow of ideas.

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