Monographs on Statistics and Applied Probability 60

Kernel Smoothing

M.P. Wand and M.C. Jones

Kernel smoothing refers to a general methodology for recovery of underlying structure in data sets. The basic principle is that local averaging or smoothing is performed with respect to a kernel function.

The main purpose of this book is to provide the uninitiated reader with a feeling for the principles, applications and analysis of kernel smoothers.This is facilitated by concentrating on the simplest settings, namely density estimation and nonparametric regression. Special attention is also given to the important problem of choosing the smoothing parameter of a kernel smoother. The multivariate case is also treated in detail. The simplicity of kernel smoothers means that the main ideas can be covered at a relatively low technical level. The book is self contained and assumes only a basic knowledge of statistics, calculus and matrix algebra. Exercises are provided at the end of each chapter.

Overall, *Kernel Smoothing* is both an invaluable introduction to the main ideas of kernel estimation for students and researchers from other disciplines and a comprehensive, up-to-date reference for those familiar with the topic.

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A complete list of the titles in this series appears in the front of the book.

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