What a wonderful book! I strongly recommend this book to anyone, especially graduate students, interested in getting a sense of 4-manifolds.

—MAA Reviews

The book gives an excellent overview of 4-manifolds, with many figures and historical notes. Graduate students, nonexperts, and experts alike will enjoy browsing through it.

— Robion C. Kirby, University of California, Berkeley

This book offers a panorama of the topology of simply connected smooth manifolds of dimension four. Dimension four is unlike any other dimension; it is large enough to have room for wild things to happen, but small enough so that there is no room to undo the wildness. For example, only manifolds of dimension four can exhibit infinitely many distinct smooth structures. Indeed, their topology remains the least understood today.

To put things in context, the book starts with a survey of higher dimensions and of topological 4-manifolds. In the second part, the main invariant of a 4-manifold—the intersection form—and its interaction with the topology of the manifold are investigated. In the third part, as an important source of examples, complex surfaces are reviewed. In the final fourth part of the book, gauge theory is presented; this differential-geometric method has brought to light how unwieldy smooth 4-manifolds truly are, and while bringing new insights, has raised more questions than answers.

The structure of the book is modular, organized into a main track of about two hundred pages, augmented by extensive notes at the end of each chapter, where many extra details, proofs and developments are presented. To help the reader, the text is peppered with over 250 illustrations and has an extensive index.

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